Summer 2015 Examination

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		Important Instructions to the Examiners: 1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme. 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate. 3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.) 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn. 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer. 6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding. 7) For programming language papers, credit may be given to any other program based on equivalent concept. -- -- **Important Note** In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. **In such case, FIRST SEE whether the method falls within the scope of the curriculum**, and THEN ONLY give appropriate marks in accordance with the scheme of marking. -- --		

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1)		**Attempt any TEN of the following:**		
	a)	Find x, if $\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & x \end{vmatrix} = 0$.		
	Ans.	$\therefore 4(-2x-28)-3(3x-77)+9(12+22)=0$	½	
		$\therefore -8x-112-9x+231+306=0$	½	
		$\therefore -17x+425=0$		
		$\therefore x=\frac{425}{17}$	½	
		$\therefore \boxed{x=25}$	½	**2**
		--		
	b)	Prove that the matrix $\begin{bmatrix} 1 & 4 \\ 6 & 9 \end{bmatrix}$ is a nonsingular matrix.		
	Ans.	$\therefore \begin{vmatrix} 1 & 4 \\ 6 & 9 \end{vmatrix} = 9-24$		
		$=-15$	1	
		$\neq 0$	½	
		Given matrix is non-singular.	½	**2**
		--		
	c)	If $A=\begin{bmatrix} 3 & 4 & -2 \\ 2 & 1 & 0 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 0 & 2 \end{bmatrix}$, find AB.		
	Ans.	$\therefore AB=\begin{bmatrix} 3 & 4 & -2 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 0 & 2 \end{bmatrix}$		
		$=\begin{bmatrix} 6+12-0 & -3+16-4 \\ 4+3+0 & -2+4+0 \end{bmatrix}$	1	
		$=\begin{bmatrix} 18 & 9 \\ 7 & 2 \end{bmatrix}$	1	**2**
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1)	d)	Resolve into partial fractions $\frac{1}{x^3-x}$.		
	Ans.	$\frac{1}{x^3-x}=\frac{1}{x(x+1)(x-1)}$ $=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$ $\therefore \boxed{1=(x+1)(x-1)A+x(x-1)B+x(x+1)C}$ $Put\ x=0$ $\therefore 1=(0+1)(0-1)A+0+0$ $\therefore 1=-A$ $\therefore \boxed{-1=A}$	½	
		$Put\ x=-1$ $\therefore 1=0-1(-1-1)B+0$ $\therefore 1=2B$ $\therefore \boxed{\frac{1}{2}=B}$	½	
		$Put\ x=1$ $\therefore 1=0+0+1(1+1)C$ $\therefore 1=2C$ $\therefore \boxed{\frac{1}{2}=C}$	½	
		$\therefore \boxed{\frac{1}{x^3-x}=\frac{-1}{x}+\frac{\frac{1}{2}}{x+1}+\frac{\frac{1}{2}}{x-1}}$	½	**2**
		--		
	e)	Define compound angle.		
	Ans.	**Compound Angle:** An angle formed by sum or difference of many angles is said to be compound angle.	2	**2**
		Note: The above definition is a sample format. Students may express the same into other words also. Please give due credit to the students.		
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1)	f)	Prove that $\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$		
	Ans.	$\sin\left(\frac{\pi}{2}+\theta\right)=\sin\frac{\pi}{2}\cos\theta+\cos\frac{\pi}{2}\sin\theta$	½	
		$=\cos\theta+0$	1	
		$=\cos\theta$	½	**2**
	g)	Express $4\cos 30^\circ\sin 20^\circ$ as the sum or difference of trignometric ratios.		
	Ans.	$4\cos 30^\circ\sin 20^\circ=2\left(2\cos 30^\circ\sin 20^\circ\right)$		
		$=2\left[\sin\left(30^\circ+20^\circ\right)-\sin\left(30^\circ-20^\circ\right)\right]$	1	
		$=2\left[\sin 50^\circ-\sin 10^\circ\right]$	1	**2**
	h)	Find the principal value of $\cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]$		
	Ans.	$\cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]=\cos\left[\frac{\pi}{2}-\frac{\pi}{6}\right]$	½	
		$=\cos\left[\frac{\pi}{3}\right]$	½	
		$=\frac{1}{2}$ *or* 0.5	1	**2**
		OR		
		$\cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]=\cos\left[90^\circ-30^\circ\right]$	½	
		$=\cos\left[60^\circ\right]$	½	
		$=\frac{1}{2}$ *or* 0.5	1	**2**
		OR		
		$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$	½	
		$\therefore\cos\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]=\cos\left[\frac{\pi}{2}-\frac{\pi}{6}\right]$		
		$=\cos\left[\frac{\pi}{3}\right]$	½	
		$=\frac{1}{2}$ *or* 0.5	1	**2**

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1)	i)	Show that the lines $5x+6y-1=0$ and $6x-5y+3=0$ are perpendicular.		
	Ans.	$i)\ For\ the\ line\ 5x+6y-1=0$		
		$\therefore slope\ m_1=-\frac{A}{B}=-\frac{5}{6}$	½	
		$ii)\ For\ the\ line\ 6x-5y+3=0$		
		$\therefore slope\ m_2=-\frac{A}{B}=-\frac{6}{-5}=\frac{6}{5}$	½	
		$\therefore m_1=-\frac{5}{6}=-\frac{1}{6/5}=-\frac{1}{m_2}$	½	
		$\therefore$ the lines are perpendicular.	½	**2**
		OR	OR	
		$\therefore m_1\cdot m_2=-\frac{5}{6}\times\frac{6}{5}=-1$	½	
		$\therefore$ the lines are perpendicular.	½	**2**
	j)	Find the equation of straight line passing through $(4,-5)$ and having slope $-\frac{2}{3}$.		
	Ans.	$\therefore$ the equation is $y-y_1=m(x-x_1)$		
		$\therefore y+5=-\frac{2}{3}(x-4)$	1	
		$\therefore 3y+15=-2x+8$	½	
		$\therefore 2x+3y+7=0$	½	**2**
	k)	Find the range and coefficient of range of the following distribution:		

x_i	10	20	30	40	50
f_i	7	5	3	2	1

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1)	Ans.	$Range = \text{Largest Value} - \text{Smallest Value}$		
		$= 50 - 10$	½	
		$= 40$	½	
		$Coefficient\ of\ Range = \dfrac{\text{Largest Value} - \text{Smallest Value}}{\text{Largest Value} + \text{Smallest Value}}$		
		$= \dfrac{50-10}{50+10}$	½	
		$= \dfrac{2}{3}$	½	**2**
		--		
	l)	If the mean is 82.5, standard deviation is 7.2, find the coefficient of variance.		
	Ans.	$Coeff.\ of\ Variance = \dfrac{S.D.}{\overline{x}} \times 100$		
		$= \dfrac{7.2}{82.5} \times 100$	1	
		$= 8.727$	1	**2**
		--		
2)		**Attempt any four of the following:**		
	a)	Solve the following equations by using Crammer's rule: $x + y = 4 - z, \quad y + z = 1 - 2x, \quad x + z = y$		
	Ans.	$x + y + z = 4$ $2x + y + z = 1$ $x - y + z = 0$		
		$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 1(1+1) - 1(2-1) + 1(-2-1)$		
		$= -2$	1	
		$D_x = \begin{vmatrix} 4 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & -1 & 1 \end{vmatrix} = 4(1+1) - 1(1-0) + 1(-1-0)$		
		$= 6$	½	

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2)		$D_y = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0)-4(2-1)+1(0-1)$		
		$= -4$	½	
		$D_z = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 1(0+1)-1(0-1)+4(-2-1)$	½	
		$= -10$		
		$\therefore x = \frac{D_x}{D} = \frac{6}{-2} = -3$	½	
		$y = \frac{D_y}{D} = \frac{-4}{-2} = 2$	½	
		$z = \frac{D_z}{D} = \frac{-10}{-2} = 5$	½	**4**
		--		
	b)	Find the matrix X such that $\begin{bmatrix} 4 & 5 \\ -3 & 6 \end{bmatrix} + X = \begin{bmatrix} 10 & -1 \\ 0 & -6 \end{bmatrix}$		
	Ans.	$\therefore X = \begin{bmatrix} 10 & -1 \\ 0 & -6 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ -3 & 6 \end{bmatrix}$	1	
		$= \begin{bmatrix} 10-4 & -1-5 \\ 0+3 & -6-6 \end{bmatrix}$	1	
		$= \begin{bmatrix} 6 & -6 \\ 3 & -12 \end{bmatrix}$	2	**4**
		OR		
		$Let \;\; X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$		
		$\therefore \begin{bmatrix} 4 & 5 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 10 & -1 \\ 0 & -6 \end{bmatrix}$		
		$\therefore \begin{bmatrix} 4+a & 5+b \\ -3+c & 6+d \end{bmatrix} = \begin{bmatrix} 10 & -1 \\ 0 & -6 \end{bmatrix}$	1	
		$\therefore 4+a=10 \quad 5+b=-1$ $-3+c=0 \quad 6+d=-6$	1	
		$\therefore a=6 \quad b=-6$ $c=3 \quad d=-12$	1	
		$\therefore X = \begin{bmatrix} 6 & -6 \\ 3 & -12 \end{bmatrix}$	1	**4**

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2)	c)	If $A=\begin{bmatrix}1 & -2\\ -3 & -1\end{bmatrix}$, $B=\begin{bmatrix}4 & 2 & -5\\ 1 & 0 & 3\end{bmatrix}$, $C=\begin{bmatrix}6 & -7 & 0\\ -1 & 2 & 5\\ 1 & 0 & 3\end{bmatrix}$, prove that $(AB)C=A(BC)$		
	Ans.	$A=\begin{bmatrix}1 & -2\\ -3 & -1\end{bmatrix}$, $B=\begin{bmatrix}4 & 2 & -5\\ 1 & 0 & 3\end{bmatrix}$, $C=\begin{bmatrix}6 & -7 & 0\\ -1 & 2 & 5\\ 1 & 0 & 3\end{bmatrix}$		
		$AB=\begin{bmatrix}1 & -2\\ -3 & -1\end{bmatrix}\begin{bmatrix}4 & 2 & -5\\ 1 & 0 & 3\end{bmatrix}$		
		$=\begin{bmatrix}4-2 & 2-0 & -5-6\\ -12-1 & -6-0 & 15-3\end{bmatrix}$	½	
		$=\begin{bmatrix}2 & 2 & -11\\ -13 & -6 & 12\end{bmatrix}$	½	
		$(AB)C=\begin{bmatrix}2 & 2 & -11\\ -13 & -6 & 12\end{bmatrix}\begin{bmatrix}6 & -7 & 0\\ -1 & 2 & 5\\ 1 & 0 & 3\end{bmatrix}$		
		$=\begin{bmatrix}12-2-11 & -14+4-0 & 0+10-33\\ -78+6+12 & 91-12+0 & 0-30+36\end{bmatrix}$	½	
		$=\begin{bmatrix}-1 & -10 & -23\\ -60 & 79 & 6\end{bmatrix}$	½	
		$BC=\begin{bmatrix}4 & 2 & -5\\ 1 & 0 & 3\end{bmatrix}\begin{bmatrix}6 & -7 & 0\\ -1 & 2 & 5\\ 1 & 0 & 3\end{bmatrix}$		
		$=\begin{bmatrix}24-2-5 & -28+4-0 & 0+10-15\\ 6-0+3 & -7+0+0 & 0+0+9\end{bmatrix}$	½	
		$=\begin{bmatrix}17 & -24 & -5\\ 9 & -7 & 9\end{bmatrix}$	½	
		$A(BC)=\begin{bmatrix}1 & -2\\ -3 & -1\end{bmatrix}\begin{bmatrix}17 & -24 & -5\\ 9 & -7 & 9\end{bmatrix}$		
		$=\begin{bmatrix}17-18 & -24+14 & -5-18\\ -51-9 & 72+7 & 15-9\end{bmatrix}$		
		$=\begin{bmatrix}-1 & -10 & -23\\ -60 & 79 & 6\end{bmatrix}$	½	
		$\therefore (AB)C=A(BC)$	½	**4**
		OR		

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2)		$(AB)C=\left\{\begin{bmatrix}1 & -2\\-3 & -1\end{bmatrix}\begin{bmatrix}4 & 2 & -5\\1 & 0 & 3\end{bmatrix}\right\}\begin{bmatrix}6 & -7 & 0\\-1 & 2 & 5\\1 & 0 & 3\end{bmatrix}$		
		$=\left\{\begin{bmatrix}4-2 & 2-0 & -5-6\\-12-1 & -6-0 & 15-3\end{bmatrix}\right\}\begin{bmatrix}6 & -7 & 0\\-1 & 2 & 5\\1 & 0 & 3\end{bmatrix}$	½	
		$=\begin{bmatrix}2 & 2 & -11\\-13 & -6 & 12\end{bmatrix}\begin{bmatrix}6 & -7 & 0\\-1 & 2 & 5\\1 & 0 & 3\end{bmatrix}$	½	
		$=\begin{bmatrix}12-2-11 & -14+4-0 & 0+10-33\\-78+6+12 & 91-12+0 & 0-30+36\end{bmatrix}$	½	
		$=\begin{bmatrix}-1 & -10 & -23\\-60 & 79 & 6\end{bmatrix}$	½	
		$A(BC)=\begin{bmatrix}1 & -2\\-3 & -1\end{bmatrix}\left\{\begin{bmatrix}4 & 2 & -5\\1 & 0 & 3\end{bmatrix}\begin{bmatrix}6 & -7 & 0\\-1 & 2 & 5\\1 & 0 & 3\end{bmatrix}\right\}$		
		$=\begin{bmatrix}1 & -2\\-3 & -1\end{bmatrix}\left\{\begin{bmatrix}24-2-5 & -28+4-0 & 0+10-15\\6-0+3 & -7+0+0 & 0+0+9\end{bmatrix}\right\}$	½	
		$=\begin{bmatrix}1 & -2\\-3 & -1\end{bmatrix}\begin{bmatrix}17 & -24 & -5\\9 & -7 & 9\end{bmatrix}$	½	
		$=\begin{bmatrix}17-18 & -24+14 & -5-18\\-51-9 & 72+7 & 15-9\end{bmatrix}$		
		$=\begin{bmatrix}-1 & -10 & -23\\-60 & 79 & 6\end{bmatrix}$	½	
		$\therefore (AB)C=A(BC)$	½	**4**
		---		
	d)	Express the matrix A as sum of symmetric and skew-symmetric matrices, if $A=\begin{bmatrix}-1 & 7 & 1\\2 & 3 & 4\\5 & 0 & 5\end{bmatrix}$		
	Ans.	$\therefore A'=\begin{bmatrix}-1 & 2 & 5\\7 & 3 & 0\\1 & 4 & 5\end{bmatrix}$	1	

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2)		$\therefore A+A'=\begin{bmatrix}-1&7&1\\2&3&4\\5&0&5\end{bmatrix}+\begin{bmatrix}-1&2&5\\7&3&0\\1&4&5\end{bmatrix}$		
		$=\begin{bmatrix}-2&9&6\\9&6&4\\6&4&10\end{bmatrix}$	1	
		$\therefore A-A'=\begin{bmatrix}-1&7&1\\2&3&4\\5&0&5\end{bmatrix}-\begin{bmatrix}-1&2&5\\7&3&0\\1&4&5\end{bmatrix}$		
		$=\begin{bmatrix}0&5&-4\\-5&0&4\\4&-4&0\end{bmatrix}$	1	
		$\therefore A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$		
		$=\frac{1}{2}\begin{bmatrix}-2&9&6\\9&6&4\\6&4&10\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0&5&-4\\-5&0&4\\4&-4&0\end{bmatrix}$	1	**4**
		OR		
		$\therefore A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$		
		$=\frac{1}{2}\left\{\begin{bmatrix}-1&7&1\\2&3&4\\5&0&5\end{bmatrix}+\begin{bmatrix}-1&2&5\\7&3&0\\1&4&5\end{bmatrix}\right\}+\frac{1}{2}\left\{\begin{bmatrix}-1&7&1\\2&3&4\\5&0&5\end{bmatrix}-\begin{bmatrix}-1&2&5\\7&3&0\\1&4&5\end{bmatrix}\right\}$	1+1	
		$=\frac{1}{2}\begin{bmatrix}-2&9&6\\9&6&4\\6&4&10\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0&5&-4\\-5&0&4\\4&-4&0\end{bmatrix}$	1+1	**4**
		--		
	e)	Resolve into partial fractions $\frac{x+5}{x^2-x}$		
	Ans.	$\frac{x+5}{x^2-x}=\frac{x+5}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}$	1	
		$\therefore \boxed{x+5=(x-1)A+xB}$		

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2)		$Put\ x=0$ $\therefore 0+5=(0-1)A+0$ $\therefore 5=-A$ $\therefore \boxed{-5=A}$	1	
		$Put\ x-1=0 \quad \therefore x=1$ $\therefore 1+5=0A+B$ $\therefore \boxed{6=B}$	1	
		$\therefore \boxed{\frac{x+5}{x^2-x}=\frac{-5}{x}+\frac{6}{x-1}}$	1	**4**
		Note for partial fraction problems: The problems of partial fractions could also be solved by the method of "equating equal power coefficients". This method is also applicable. Give appropriate marks in accordance with the scheme of marking in the later/previous problems as the solution by this method is not discussed. For the sake of convenience, the solution of the above problem with the help of this method is illustrated hereunder.		
		$\frac{x+5}{x^2-x}=\frac{x+5}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}$	1	
		$\therefore x+5=(x-1)A+xB$ $\therefore x+5=xA-A+xB$ $\therefore 1\cdot x+5=(A+B)x-A$ $\therefore A+B=1, \quad -A=5$ $\therefore \boxed{A=-5}$	1	
		$\therefore B=1-A=1+5$ $\therefore \boxed{B=6}$	1	
		$\therefore \boxed{\frac{1}{x^2-x}=\frac{-5}{x}+\frac{6}{x-1}}$	1	**4**
		Note: The above problem can also be solved as follows: $\frac{x+1}{x^2-x}=\frac{x+1}{(x-1)x}=\frac{A}{x-1}+\frac{B}{x}$ In this case, we get $\therefore \boxed{A=6} \quad \boxed{B=-5}$ $\therefore \boxed{\frac{x+1}{x^2-x}=\frac{6}{x-1}+\frac{-5}{x}}$		

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2)	f)	Resolve into partial fractions $\dfrac{x^2+36x+6}{(x-1)(x^2+2)}$		
	Ans.	$\dfrac{x^2+36x+6}{(x-1)(x^2+2)}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+2}$	½	
		$\therefore \boxed{x^2+36x+6=(x^2+2)A+(x-1)(Bx+C)}$		
		$Put\ \ x-1=0\quad i.e.,\ x=1$		
		$\therefore 1^2+36(1)+6=A(1^2+2)+0$		
		$\therefore 43=3A$		
		$\therefore \boxed{\dfrac{43}{3}=A}$	1	
		$Put\ \ x=0$		
		$\therefore 0+0+6=(0+2)A+(0-1)(0+C)$		
		$\therefore 6=2A-C$		
		$\therefore 6-2A=-C$		
		$\therefore 6-2\left(\dfrac{43}{3}\right)=-C$		
		$\therefore -\dfrac{68}{3}=-C$		
		$\therefore \boxed{\dfrac{68}{3}=C}$	1	
		$Put\ \ x=2$		
		$\therefore 2^2+36(2)+6=(2^2+2)A+(2-1)(2B+C)$		
		$\therefore 82=6A+2B+C$		
		$\therefore 82-6A-C=2B$		
		$\therefore 82-6\left(\dfrac{43}{3}\right)-\dfrac{68}{3}=2B$		
		$\therefore -\dfrac{80}{3}=2B$		
		$\therefore \boxed{-\dfrac{40}{3}=B}$	1	
		$\therefore \boxed{\dfrac{x^2+36x+6}{(x-1)(x^2+2)}=\dfrac{\frac{43}{3}}{x-1}+\dfrac{-\frac{40}{3}x+\frac{68}{3}}{x^2+2}}$	½	**4**

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3)		**Attempt any four of the following:**		
	a)	Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 4 \\ -1 & 2 & 3 \\ 1 & 4 & 1 \end{bmatrix}$		
	Ans.	$Let\ \ A=\begin{bmatrix} 1 & 2 & 4 \\ -1 & 2 & 3 \\ 1 & 4 & 1 \end{bmatrix}$		
		$\therefore \lvert A \rvert = 1(2-12)-2(-1-3)+4(-4-2)=-26$ $\therefore A^{-1}\ exists.$	½	
		Matrix of Cofactor of A is,		
		$C(A)=\begin{bmatrix} \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} \\ -\begin{vmatrix} 2 & 4 \\ 4 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} \\ \begin{vmatrix} 2 & 4 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 4 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} \end{bmatrix}$	½	
		$=\begin{bmatrix} -10 & 4 & -6 \\ 14 & -3 & -2 \\ -2 & -7 & 4 \end{bmatrix}$ ---(*)	1½	
		$adj(A)=\begin{bmatrix} -10 & 14 & -2 \\ 4 & -3 & -7 \\ -6 & -2 & 4 \end{bmatrix}$	½	
		$\therefore A^{-1}=\frac{1}{\lvert A \rvert} adj(A)$ $=\frac{1}{-26}\begin{bmatrix} -10 & 14 & -2 \\ 4 & -3 & -7 \\ -6 & -2 & 4 \end{bmatrix}$	1	**4**
		(*) Note: In the matrix *C(A)*, if 1 to 3 elements are wrong (either in sign or value), deduct ½ mark, if 4 to 6 elements are wrong, deduct 1½ marks, if 7 to 9 are wrong, deduct all the 2 marks. Further, if all the elements in the last i.e., *adj (A)* are correct, then only give ½ mark.		
		OR	OR	

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3)		Matrix of minors of A is,		
		$M\left(A\right)=\begin{bmatrix} \begin{vmatrix}2&3\\4&1\end{vmatrix} & \begin{vmatrix}-1&3\\1&1\end{vmatrix} & \begin{vmatrix}-1&2\\1&4\end{vmatrix} \\ \begin{vmatrix}2&4\\4&1\end{vmatrix} & \begin{vmatrix}1&4\\1&1\end{vmatrix} & \begin{vmatrix}1&2\\1&4\end{vmatrix} \\ \begin{vmatrix}2&4\\2&3\end{vmatrix} & \begin{vmatrix}1&4\\-1&3\end{vmatrix} & \begin{vmatrix}1&2\\-1&2\end{vmatrix} \end{bmatrix}$	½	
		$=\begin{bmatrix}-10&-4&-6\\-14&-3&2\\-2&7&4\end{bmatrix} \quad ---(*)$	1	
		$C\left(A\right)=\begin{bmatrix}-10&4&-6\\14&-3&-2\\-2&-7&4\end{bmatrix}$	½	**4**
		OR	OR	
		$A_{11}=\begin{vmatrix}2&3\\4&1\end{vmatrix}=-10 \quad A_{12}=-\begin{vmatrix}-1&3\\1&1\end{vmatrix}=4 \quad A_{13}=\begin{vmatrix}-1&2\\1&4\end{vmatrix}=-6$ $A_{21}=-\begin{vmatrix}2&4\\4&1\end{vmatrix}=14 \quad A_{22}=\begin{vmatrix}1&4\\1&1\end{vmatrix}=-3 \quad A_{23}=-\begin{vmatrix}1&2\\1&4\end{vmatrix}=-2$ $A_{31}=\begin{vmatrix}2&4\\2&3\end{vmatrix}=-2 \quad A_{32}=-\begin{vmatrix}1&4\\-1&3\end{vmatrix}=-7 \quad A_{33}=\begin{vmatrix}1&2\\-1&2\end{vmatrix}=4$	1½	
		$\therefore C\left(A\right)=\begin{bmatrix}-10&4&-6\\14&-3&-2\\-2&-7&4\end{bmatrix}$	½	**4**
		Note: In the above, if 1 to 3 elements are wrong, deduct ½ mark, if 4 to 6 elements are wrong, deduct 1 marks, and if 7 to 9 are wrong, deduct all the marks. Further, if all the elements in the following matrices C(A) and adj (A) are correct, then only give the marks. ---		

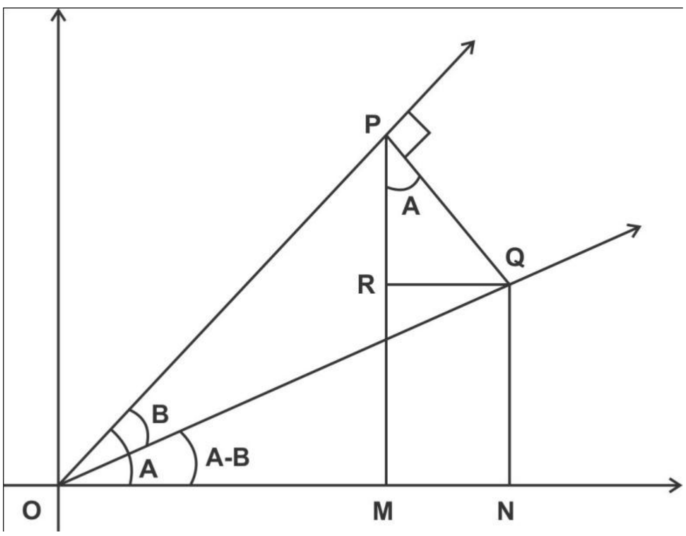
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	b)	Solve by matrix method: $3x+y+2z=3,\quad 2x-3y-z=-3,\quad x+2y+z=4$		
	Ans.	$\therefore A=\begin{bmatrix}3&1&2\\2&-3&-1\\1&2&1\end{bmatrix},\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix},\quad B=\begin{bmatrix}3\\-3\\4\end{bmatrix}$		
		$\therefore \lvert A\rvert=3(-3+2)-1(2+1)+2(4+3)$ $=8$	½	
		$\therefore adj(A)=\begin{bmatrix}-1&3&5\\-3&1&7\\7&-5&-11\end{bmatrix}$ -----------(*)	1	
		$\therefore A^{-1}=\dfrac{1}{\lvert A\rvert}adj(A)$ $=\dfrac{1}{8}\begin{bmatrix}-1&3&5\\-3&1&7\\7&-5&-11\end{bmatrix}$	1	
		$\therefore$ the solution is, $X=A^{-1}B$ $=\dfrac{1}{8}\begin{bmatrix}-1&3&5\\-3&1&7\\7&-5&-11\end{bmatrix}\begin{bmatrix}3\\-3\\4\end{bmatrix}$ $=\dfrac{1}{8}\begin{bmatrix}8\\16\\-8\end{bmatrix}$ $=\begin{bmatrix}1\\2\\-1\end{bmatrix}$	1	
		$\therefore x=1,\quad y=2,\quad z=-1$	½	**4**
		(*) Note: Many other methods are followed to find $adj(A)$ as discussed in the Q. 3 (a). Please give appropriate marks in accordance with the scheme of marking as discussed therein. --		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	c)	Resolve into partial fractions $\frac{x^3+1}{x^2+6x}$		
	Ans.	$\frac{x^3+1}{x^2+6x}=x-6+\frac{36x+1}{x^2+6x}$	1	
		$\therefore \frac{36x+1}{x^2+6x}=\frac{36x+1}{x(x+6)}=\frac{A}{x}+\frac{B}{x+6}$	½	
		$\therefore \boxed{36x+1=(x+6)A+xB}$		
		$Put\ x=0$		
		$\therefore 0+1=(0+6)A+0$		
		$\therefore 1=6A$		
		$\therefore \boxed{\frac{1}{6}=A}$	1	
		$Put\ x+6=0 \quad i.e.,\ x=-6$		
		$\therefore 36(-6)+1=0-6B$		
		$\therefore -215=-6B$		
		$\therefore \boxed{\frac{215}{6}=B}$	½	
		$\therefore \frac{x^3+1}{x^2+6x}=\frac{\frac{1}{6}}{x}+\frac{\frac{215}{6}}{x+2}$	½	
		$\therefore \boxed{\frac{x^3+1}{x^2+6x}=x-6+\frac{\frac{1}{6}}{x}+\frac{\frac{215}{6}}{x+2}}$	½	**4**
		---		
	d)	Resolve into partial fractions $\frac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)}$		
	Ans.	$Put\ \tan\theta=x$		
		$\frac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)}=\frac{x+1}{(x+2)(x+3)}=\frac{A}{x+2}+\frac{B}{x+3}$	1	
		$\therefore \boxed{x+1=(x+3)A+(x+2)B}$		
		$Put\ x=-2$		
		$\therefore -2+1=(-2+3)A+0$		
		$\therefore \boxed{A=-1}$	1	

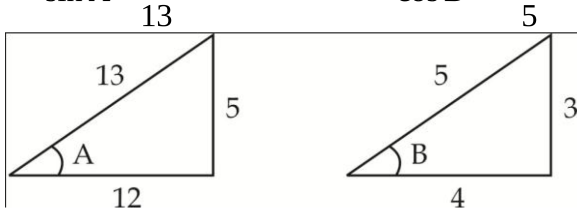
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$Put\ x=-3$ $\therefore -3+1=0+(-3+2)B$ $\therefore \boxed{B=2}$ $\therefore \dfrac{x+1}{(x+2)(x+3)}=\dfrac{-1}{x+2}+\dfrac{2}{x+3}$ $\therefore \boxed{\dfrac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)}=\dfrac{-1}{\tan\theta+2}+\dfrac{2}{\tan\theta+3}}$	1 ½ ½	**4**
		OR		
		$\dfrac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)}=\dfrac{A}{\tan\theta+2}+\dfrac{B}{\tan\theta+3}$ $\therefore \boxed{\tan\theta+1=(\tan\theta+3)A+(\tan\theta+2)B}$ $Put\ \tan\theta=-2$ $\therefore -2+1=(-2+3)A+0$ $\therefore \boxed{A=-1}$ $Put\ \tan\theta=-3$ $\therefore -3+1=0+(-3+2)B$ $\therefore \boxed{B=2}$ $\therefore \boxed{\dfrac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)}=\dfrac{-1}{\tan\theta+2}+\dfrac{2}{\tan\theta+3}}$	1 1 1 1	**4**
		--		
	e)	If $\cos A=-\dfrac{3}{5},\ \sin B=\dfrac{20}{29}$, where A and B are the angles in the third and second quadrant respectively, find $\tan(A+B)$.		
	Ans.	$\cos A=-\dfrac{3}{5}$ $\qquad$ $\sin B=\dfrac{20}{29}$  		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		As A is the third quadrant, tan A is positive and B is in the second quadrant, tan B is negative.		
		$\therefore \tan A = \frac{4}{3} \quad and \quad \tan B = -\frac{20}{21}$	1+1	
		$\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$		
		$= \frac{\frac{4}{3} - \frac{20}{21}}{1 - \left(\frac{4}{3}\right)\left(-\frac{20}{21}\right)} \quad ---(*)$	1	
		$= \frac{24}{143} \quad or \quad 0.168 \quad ---(**)$	1	**4**
		Note (*): Due to the use of advance scientific calculators which is permissible in the exam, students may write the step (**) directly after step (*). Writing such step is to be considered.		
		Note: To evaluate value of tan A and tan B, many times the relations between sine ratio and cosine ratio is used (as illustrated hereunder), instead of using Triangle Method as illustrated in the above solution. As the main object is to find the values, please consider these methods also.		
		$\tan A = \frac{\sqrt{1-\cos^2 A}}{\cos A} = \frac{\sqrt{1-\frac{9}{25}}}{3/5} = \frac{4}{5}$		
		$\tan B = -\frac{\sin B}{\sqrt{1-\sin^2 B}} = -\frac{20/29}{\sqrt{1-\frac{400}{841}}} = -\frac{20}{21}$		
		---		
	f)	Without using calculator find the value of $\sin(150^o) - \tan(315^o) + \cos(300^o) + \sec^2(360^o)$		
	Ans.	$\sin(150^o) = \sin(90^o + 60^o) \quad or \quad \sin(180^o - 30^o)$		
		$= \cos 60^o \quad or \quad \sin 30^o$	½	
		$= \frac{1}{2}$	½	

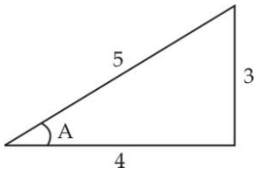
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$\tan(315^o) = \tan(270^o + 45^o)$ *or* $\tan(360^o - 45^o)$		
		$= \tan(3\times 90^o + 45^o)$ *or* $\tan(4\times 90^o - 45^o)$	½	
		$= -\cot 45^o$ *or* $-\tan 45^o$	½	
		$= -1$		
		$\cos(300^o) = \cos(270^o + 30^o)$ *or* $\cos(360^o - 60^o)$		
		$= \cos(3\times 90^o + 30^o)$ *or* $\cos(4\times 90^o - 60^o)$		
		$= \sin 30^o$ *or* $\cos 60^o$	½	
		$= \frac{1}{2}$	½	
		$\sec^2(360^o) = \sec^2(0^o) = 1$	½	
		$\therefore \sin(150^o) - \tan(315^o) + \cos(300^o) + \sec^2(360^o)$		
		$= \frac{1}{2} + 1 + \frac{1}{2} + 1$		**4**
		$= 3$	½	
		OR		
		By taking the various combinations of the above different trigonometric ratios, students may solve the problem as per his/her convenience by taking all the ratios together. One of them is illustrated for the sake of scheme of marking:		
		$\therefore \sin(150^o) - \tan(315^o) + \cos(300^o) + \sec^2(360^o)$		
		$= \sin(90^o + 60^o) - \tan(360^o - 45^o) + \cos(270^o + 30^o) + \sec^2(0^o)$		
		$= \cos 60^o - \tan(4\times 90^o - 45^o) + \cos(3\times 90^o + 30^o) + \sec^2(0^o)$		
		$= \cos 60^o + \tan 45^o + \sin 30^o + \sec^2(0^o)$	½+½+ ½	
		$= \frac{1}{2} + 1 + \frac{1}{2} + 1$	½+½+ ½+½	
		$= 3$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**Attempt any four of the following:**		
	a)	Prove that $1+\tan A\cdot\tan 2A=\sec 2A$		
	Ans.	$1+\tan A\cdot\tan 2A=1+\frac{\sin A}{\cos A}\times\frac{\sin 2A}{\cos 2A}$	1	
		$=\frac{\cos A\cos 2A+\sin A\sin 2A}{\cos A\cos 2A}$		
		$=\frac{\cos(A-2A)}{\cos A\cos 2A}$	1	
		$=\frac{\cos(-A)}{\cos A\cos 2A}$		
		$=\frac{\cos A}{\cos A\cos 2A}$	1	
		$=\sec 2A$	1	**4**
		OR		
		$1+\tan A\cdot\tan 2A=1+\tan A\left(\frac{2\tan A}{1-\tan^2 A}\right)$	1	
		$=\frac{1-\tan^2 A+2\tan^2 A}{1-\tan^2 A}$	1	
		$=\frac{1+\tan^2 A}{1-\tan^2 A}$	1	
		$=\sec 2A$	1	**4**
		--		
	b)	Prove that $\sin(A-B)=\sin A\cos B-\cos A\sin B$		
	Ans.		1	

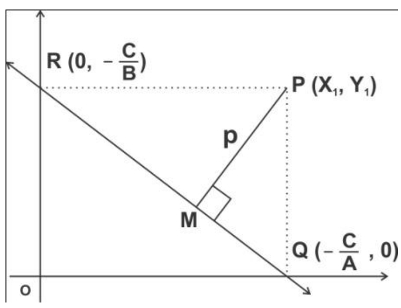
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\sin(A-B)=\frac{NQ}{OQ}$ $=\frac{RM}{OQ}$ $=\frac{PM-PR}{OQ}$ $=\frac{PM}{OQ}-\frac{PR}{OQ}$ $=\frac{PM}{OP}\times\frac{OP}{OQ}-\frac{PR}{PQ}\times\frac{PQ}{OQ}$ $=\sin A\cos B-\cos A\sin B$	 1 1 1	4
		Note: The above is proved by different ways in several books. Consider all these proof but check whether the method is falling within the scope of curriculum and give appropriate marks in accordance with the scheme of marking. In accordance with the Teacher's Manual published by MSBTE, the result is treated as Fundamental Result which is not proved by the help of any another result. If the above result is proved by students using any another result, suppose using sin (A+B), then this result i.e., sin (A+B) must have been proved first. ---		
	c)	If A and B both are obtuse angles and $\sin A=\frac{5}{13},\ \cos B=-\frac{4}{5}$, find the quadrant of the angle $A+B$.		
	Ans.	$\sin A=\frac{5}{13},\ \cos B=-\frac{4}{5}$ As A and B are abtuse angles, $\cos A$ is negative and $\sin B$ is positive. $\therefore\cos A=-\sqrt{1-\sin^2 A}=-\sqrt{1-\left(\frac{5}{13}\right)^2}=-\frac{12}{13}$ $\sin B=+\sqrt{1-\cos^2 B}=+\sqrt{1-\left(-\frac{4}{5}\right)^2}=\frac{3}{5}$ $\therefore\cos(A+B)=\cos A\cos B-\sin A\sin B$ $=-\frac{12}{13}\cdot\left(-\frac{4}{5}\right)-\frac{5}{13}\cdot\frac{3}{5}$ ---(*) $=\frac{33}{65}$ ---(*)	 ½ ½ 1 1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		As A and B are abtuse angles, $180^\circ < A+B < 360^\circ$.		
		In III quadrant, cos is -ve.		
		$\therefore A+B$ is in IV quadrant.	1	
		OR		
		$\sin A = \frac{5}{13}$ $\cos B = -\frac{4}{5}$		
				
		As A and B are abtuse angles, $\cos A$ is negative and $\sin B$ is positive.		
		$\therefore \cos A = -\frac{12}{13}$	½	
		$\sin B = \frac{3}{5}$	½	
		$\therefore \cos(A+B) = \cos A \cos B - \sin A \sin B$		
		$= -\frac{12}{13}\cdot\left(-\frac{4}{5}\right) - \frac{5}{13}\cdot\frac{3}{5}$ $---(*)$	1	
		$= \frac{33}{65}$ $---(*)$	1	
		As A and B are abtuse angles, $180^\circ < A+B < 360^\circ$.		
		In III quadrant, cos is -ve.	1	**4**
		$\therefore A+B$ is in IV quadrant.		
		OR		
		A and B both are obtuse angles, tan A and tan B are negative.		
		$\therefore \tan A = -\frac{5}{12}$ *and* $\tan B = -\frac{3}{4}$	½+½	
		$\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$		
		$= \frac{-\frac{5}{12} - \frac{3}{4}}{1 - \left(-\frac{5}{12}\right)\left(-\frac{3}{4}\right)}$ $---(*)$	1	
		$= -\frac{56}{33}$ *or* -1.697 $---(**)$	1	
		In the III quadrant tan is +ve and in the IV quadrant, tan is –ve.	1	**4**
		$\therefore A+B$ is in the IV quadrant.		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**Note (*):** Due to the use of advance scientific calculators which is permissible in the exam, students may write the step (**) directly after step (*). Writing such step is to be considered.		
	d)	Prove that $\frac{\sin 8x-\sin 5x}{\cos 7x+\cos 6x}=\sin x+\cos x\cdot\tan\frac{x}{2}$		
	Ans.	$\frac{\sin 8x-\sin 5x}{\cos 7x+\cos 6x}=\frac{2\cos\left(\frac{8x+5x}{2}\right)\sin\left(\frac{8x-5x}{2}\right)}{2\cos\left(\frac{7x+6x}{2}\right)\cos\left(\frac{7x-6x}{2}\right)}$		
		$=\frac{2\cos\left(\frac{13x}{2}\right)\sin\left(\frac{3x}{2}\right)}{2\cos\left(\frac{13x}{2}\right)\cos\left(\frac{x}{2}\right)}$	1	
		$=\frac{\sin\left(\frac{3x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$	1	
		$=\frac{\sin\left(x+\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$		
		$=\frac{\sin x\cos\left(\frac{x}{2}\right)+\cos x\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$	1	
		$=\sin x+\cos x\cdot\tan\frac{x}{2}$	1	**4**
	e)	Prove that $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$		
	Ans.	$2\tan^{-1}x=\tan^{-1}x+\tan^{-1}x$	1	
		$=\tan^{-1}\left(\frac{x+x}{1-x.x}\right)$	2	
		$=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	f)	Prove that $\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{27}{11}\right)$		
	Ans.	*Let* $A=\cos^{-1}\left(\frac{4}{5}\right)$ $\therefore \cos A=\frac{4}{5}$ 		
		$\therefore \tan A=\frac{3}{4}$ ----------(*)	1	
		$\therefore A=\tan^{-1}\left(\frac{3}{4}\right)$ $\therefore \cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)$	1	
		$=\tan^{-1}\left(\dfrac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\cdot\frac{3}{5}}\right)$	1	
		$=\tan^{-1}\left(\frac{27}{11}\right)$	1	**4**
		Note (*): To evaluate value of tan A, various methods are used by students, such as 'using the relation between sin A and tan A' or 'first to find sin A using cos A and find tan A' etc., instead of using Triangle Method as illustrated in the above solution. As main object is to find the value of tan A, please consider these methods also. --		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		**Attempt any four of the following:**		
	a)	Prove that $\frac{\sin 4\theta+\sin 2\theta}{1+\cos 2\theta+\cos 4\theta}=\tan 2\theta$		
	Ans.	$\frac{\sin 4\theta+\sin 2\theta}{1+\cos 2\theta+\cos 4\theta}=\frac{\sin 2(2\theta)+\sin 2\theta}{1+\cos 2(2\theta)+\cos 2\theta}$	1	
		$=\frac{2\sin 2\theta\cos 2\theta+\sin 2\theta}{2\cos^2 2\theta+\cos 2\theta}$	1	
		$=\frac{\sin 2\theta(2\cos 2\theta+1)}{\cos 2\theta(2\cos 2\theta+1)}$	1	
		$=\frac{\sin 2\theta}{\cos 2\theta}$		
		$=\tan 2\theta$	1	**4**
	b)	Prove that $\frac{\sin 4A+\sin 5A+\sin 6A}{\cos 4A+\cos 5A+\cos 6A}=\tan 5A$		
	Ans.	$\frac{\sin 4A+\sin 5A+\sin 6A}{\cos 4A+\cos 5A+\cos 6A}=\frac{\sin 4A+\sin 6A+\sin 5A}{\cos 4A+\cos 6A+\cos 5A}$		
		$=\frac{2\sin 5A\cos(-A)+\sin 5A}{2\cos 5A\cos(-A)+\cos 5A}$	1+1	
		$=\frac{\sin 5A[2\cos(-A)+1]}{\cos 5A[2\cos(-A)+1]}$	1	
		$=\tan 5A$	1	**4**
	c)	Prove that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right),\quad x>0,\quad y>0,\quad xy<1$		
	Ans.	$Put\ \tan^{-1}x=A \quad and \quad \tan^{-1}y=B$ $\therefore\ x=\tan A \quad and \quad y=\tan B$		
		$\therefore \tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$	1	
		$=\frac{x+y}{1-xy}$	1	
		$\therefore A+B=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
		$\therefore \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	d)	Find the angle between the lines $y=5x+6$ and $y=x$		
	Ans.	$For\ y=5x+6\ \ i.e.,\ \ 5x-y+6=0\ \ or\ \ -5x+y-6=0$		
		$slope\ \ m_1=-\dfrac{a}{b}=5$	1	
		$For\ \ y=x\ \ or\ \ x-y=0,$		
		$slope\ \ m_1=-\dfrac{a}{b}=1$	1	
		$\therefore\tan\theta=\left\lvert\dfrac{m_1-m_2}{1+m_1\cdot m_2}\right\rvert$		
		$=\left\lvert\dfrac{5-1}{1+(5)\cdot(1)}\right\rvert$	1	
		$=\dfrac{2}{3}\ \ or\ \ 0.667$	½	
		$\therefore\theta=\tan^{-1}\left(\dfrac{2}{3}\right)\ \ or\ \ \tan^{-1}(0.667)$	½	**4**
		---		
	e)	If $P(x_1,\ y_1)$ is any point and $Ax+By+C=0$ is a line, prove that the perpendicular distance of a point P from the line is given by $\left\lvert\dfrac{Ax_1+By_1+C}{\sqrt{a^2+b^2}}\right\rvert$		
	Ans.			
		$Area\ \ of\ \Delta PQR=\dfrac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ -\dfrac{C}{A} & 0 & 1\\ 0 & -\dfrac{C}{B} & 1\end{vmatrix}$		
		$=\dfrac{C}{2AB}(Ax_1+By_1+C)$	1	
		$Now\ \ QR=\sqrt{\left(-\dfrac{C}{A}-0\right)^2+\left(0+\dfrac{C}{B}\right)^2}=\dfrac{C\sqrt{A^2+B^2}}{AB}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$But\quad Area\ \ of\ \Delta PQR = \frac{1}{2}\times AB\times PM$		
		$= \frac{1}{2}\times\frac{C\sqrt{A^2+B^2}}{AB}\times p$	1	
		$\therefore \frac{1}{2}\times\frac{C\sqrt{A^2+B^2}}{AB}\times p = \frac{C}{2AB}\left(Ax_1+By_1+C\right)$	½	
		$\therefore\ p = \frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}$	½	
		$As\ length\ is\ always\ +ve,$		
		$p = \left\vert\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right\vert$	½	**4**
		---		
	f)	Find the equation of line passing through the point of intersection of the lines $2x+3y=13,\ 5x-y=7$ and perpendicular to the line $3x-y+7=0$		
	Ans.	$2x+3y=13$ $\underline{5x-y=7}$		
		$\therefore\ 2x+3y=13$ $\underline{15x-3y=21}$		
		$\therefore\ \ 17x=34$	½	
		$\therefore x=2$ $y=3$	½	
		$\therefore$ Point of intersection $=\left(2,\ 3\right)$		
		Slope of the line $3x-y+7=0$ is,		
		$m_0 = -\frac{a}{b} = -\frac{3}{-1} = 3$	1	
		$\therefore$ Slope of the required line is,		
		$m = -\frac{1}{m_0} = -\frac{1}{3}$	1	
		$\therefore\ equation\ is,$		
		$y-y_1 = m\left(x-x_1\right)$		
		$\therefore\ y-3 = -\frac{1}{3}\left(x-2\right)$	½	
		$\therefore x+3y-11=0$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		**Attempt any four of the following:**		
	a)	Find the equation of the lines passing through the point $(6, 5)$ and parallel to the line having intercepts 2 and 4 on X and Y axis respectively.		
	Ans.	slope of given line is $m_0 = -\frac{y-\text{int}}{x-\text{int}} = -\frac{4}{2} = -2$	1 or	
		OR		
		Given line is passing through $(2, 0)$ & $(0, 4)$		
		$\therefore$ slope of given line is $m_0 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-2} = -2$	1	
		$\therefore$ Slope of the required line is,	1	
		$m = m_0 = -2$		
		$\therefore$ *equation is,*		
		$y - y_1 = m(x - x_1)$		
		$\therefore\ y - 5 = -2(x-6)$	1	
		$\therefore\ y - 5 = -2x + 12$		
		$\therefore 2x + y - 17 = 0$	1	**4**
		--		
	b)	Find the acute angle between the lines $3x - 2y + 4 = 0$ and $2x - 3y - 7 = 0$.		
	Ans.	*For* $3x - 2y + 4 = 0,$		
		slope $m_1 = -\frac{a}{b} = -\frac{3}{-2} = \frac{3}{2}$	½	
		For $2x - 3y - 7 = 0,$		
		slope $m_2 = -\frac{a}{b} = -\frac{2}{-3} = \frac{2}{3}$	½	
		$\therefore \tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right\rvert$		
		$= \left\lvert \frac{\frac{3}{2} - \frac{2}{3}}{1 + \left(\frac{3}{2}\right)\cdot\left(\frac{2}{3}\right)} \right\rvert$	1	
		$= \frac{5}{12}$ *or* 0.417	1	
		$\therefore \theta = \tan^{-1}\left(\frac{5}{12}\right)$ *or* $\tan^{-1}(0.417)$ *or* 22.636° *or* 0.395^C	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	c)	The two sets of observations are given below: **Set I**: $\bar{x}=82.5$, $\sigma=7.3$ **Set II**: $\bar{x}=98.75$, $\sigma=8.35$ Which of the two sets is more consistent?		
	Ans.	$C.V.(I)=\frac{\sigma}{\bar{x}}\times 100=\frac{7.3}{82.5}\times 100=8.848$	1	
		$C.V.(II)=\frac{\sigma}{\bar{x}}\times 100=\frac{8.35}{98.75}\times 100=8.456$	1	
		$\therefore C.V.(II)<C.V.(I)$	1	
		$\therefore$ $Set\ II$ is more consistent.	1	**4**
	d)	Find the variance and coefficient of variance of the following:		

Class Interval	55-65	65-75	75-85	85-95	95-105	105-115	115-125
Frequencies	10	12	15	20	14	7	2

Ans.

Class	xi	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
55-65	60	10	600	3600	36000
65-75	70	12	840	4900	58800
75-85	80	15	1200	6400	96000
85-95	90	20	1800	8100	162000
95-105	100	14	1400	10000	140000
105-115	110	7	770	12100	84700
115-125	120	2	240	14400	28800
		80	**6850**		**606300**

(1 mark)

$$\bar{x}=\frac{\sum f_i x_i}{N}=\frac{6850}{80}=85.625$$ (1 mark)

$$S.D.=\sqrt{\frac{\sum f_i x_i^2}{N}-\left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$=\sqrt{\frac{606300}{80}-\left(\frac{6850}{80}\right)^2}$$

$$=15.7197$$ (½ mark)

$$\therefore Variance=(S.D.)^2=15.7197^2=247.109$$ (½ mark)

$$Coeff.\ of\ Variance=\frac{S.D.}{\bar{x}}\times 100$$

$$=\frac{15.7197}{82.625}\times 100$$ (½ mark)

$$=18.359$$ (½ mark)

Total Marks: **4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		**OR**	OR	
		$\therefore Variance = \frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2$		
		$= \frac{606300}{80} - \left(\frac{6850}{80}\right)^2$	1	
		$= 247.109$		
		$Coeff.\ of\ Variance = \frac{\sqrt{Variance}}{\bar{x}} \times 100$		
		$= \frac{\sqrt{247.109}}{82.625} \times 100$	1	4
		$= 18.359$		
		OR		

Class	xi	f_i	d_i	$f_i d_i$	d_i^2	$f_i d_i^2$
55-65	60	10	-3	-30	9	90
65-75	70	12	-2	-24	4	48
75-85	80	15	-1	-15	1	15
85-95	**90**	**20**	**0**	**0**	**0**	**0**
95-105	100	14	1	14	1	14
105-115	110	7	2	14	4	28
115-125	120	2	3	6	9	18
		80		**-35**		**213**

Marks: 1

$A = 90, \quad h = 10, \quad d_i = \frac{x_i - A}{h}$

$\therefore \bar{x} = A + \frac{\sum f_i d_i}{N} \times h$

$= 90 + \frac{-35}{80} \times 10$

$= 85.625$ — 1

$S.D. = h \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$

$= 10 \times \sqrt{\frac{213}{80} - \left(\frac{-35}{80}\right)^2}$

$= 15.7197$ — ½

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$\therefore Variance = (S.D.)^2 = 15.7197^2 = 247.109$	½	
		$Coeff.\ of\ Variance = \frac{S.D.}{\bar{x}} \times 100$		
		$= \frac{15.7197}{85.625} \times 100$		
		$= 18.359$	1	**4**
		OR	OR	
		$\therefore Variance = h^2 \left[\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N} \right)^2 \right]$		
		$= 10^2 \left[\frac{213}{80} - \left(\frac{-35}{80} \right)^2 \right]$		
		$= 247.109$	1	
		$Coeff.\ of\ Variance = \frac{\sqrt{Variance}}{\bar{x}} \times 100$		
		$= \frac{\sqrt{247.109}}{82.625} \times 100$		
		$= 18.359$	1	**4**
		---		
	e)	Calculate the standard deviation of the following table:		
		Weekly expenditure Below: 05, 10, 15, 20, 25; No. of Students: 06, 16, 28, 38, 46		
	Ans.	(see table below)	1+1	
		$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N} \right)^2}$		
		$= \sqrt{\frac{9587.5}{46} - \left(\frac{595}{46} \right)^2}$	1	
		$= 6.412$	1	**4**

Weekly expenditure Below	05	10	15	20	25
No. of Students	06	16	28	38	46

Class	xi	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-5	2.5	6	15	6.25	37.5
5-10	7.5	10	75	56.25	562.5
10-15	12.5	12	150	156.25	1875
15-20	17.5	10	175	306.25	3062.5
20-25	22.5	8	180	506.25	4050
		46	**595**		**9587.5**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		**OR**		

Class	xi	f_i	d_i	$f_i d_i$	d_i^2	$f_i d_i^2$
0-5	2.5	6	-2	-12	4	24
5-10	7.5	10	-1	-10	1	10
10-15	**12.5**	**12**	**0**	**0**	**0**	**0**
15-20	17.5	10	1	10	1	10
20-25	22.5	8	2	16	4	32
		46		**4**		**76**

1+1

$$S.D. = h \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

$$= 5 \times \sqrt{\frac{76}{46} - \left(\frac{4}{46}\right)^2}$$ 1

$$= 6.412$$ 1 **4**

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f) Calculate the mean deviation about the mean of the following distribution:

xi	3	4	5	6	7	8
fi	4	9	10	8	6	3

Ans.

xi	f_i	$f_i x_i$	$D_i = \lvert x_i - \bar{x} \rvert$	$f_i D_i$
3	4	12	2.3	9.2
4	9	36	1.3	11.7
5	10	50	0.3	3
6	8	48	0.7	5.6
7	6	42	1.7	10.2
8	3	24	2.7	8.1
	40	**212**		**47.8**

1+1

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{212}{40} = 5.3$$ 1

$$M.D. = \frac{\sum f_i D_i}{N}$$

$$= \frac{47.8}{40}$$ ½

$$= 1.195$$ ½ **4**