



Summer 2015 Examination

Subject & Code: Applied Maths (17301)

Model Answer

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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		<p><b>Important Instructions to the Examiners:</b></p> <ol style="list-style-type: none"><li>1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme.</li><li>2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.</li><li>3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)</li><li>4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.</li><li>5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer.</li><li>6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.</li><li>7) For programming language papers, credit may be given to any other program based on equivalent concept.</li></ol>		





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1)	d)	Evaluate $\int xe^x dx$ .		
	Ans.	$\int xe^x dx = x \int e^x dx - \int \left[ \int e^x dx \right] \frac{d}{dx}(x) \cdot dx$ $= xe^x - \int e^x dx$ $= xe^x - e^x + c$	<p>1/2</p> <p>1/2</p> <p>1/2+1/2</p>	2
e)	Evaluate $\int \frac{1}{(x+3)(x+2)} dx$			
Ans.	$I = \int \frac{1}{(x+3)(x+2)} dx$ $\frac{1}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$ $\therefore A = -1$ $B = 1 \quad \text{-----} (*)$ $\frac{1}{(x+3)(x+2)} = \frac{-1}{x+3} + \frac{1}{x+2}$ $\therefore I = \int \left[ \frac{-1}{x+3} + \frac{1}{x+2} \right] dx$ $= -\log(x+3) + \log(x+2) + c$	<p>1/2</p> <p>1/2</p> <p>1/2+1/2</p>	2	
<p><b>Note (*):</b> There are various methods to find the values of A and B to partially factorize the given expression including direct method. Students may apply any one of the methods. Take in count all such methods.</p>				
<b>OR</b>				
		$I = \int \frac{1}{(x+3)(x+2)} dx$ $= \int \frac{1}{x^2 + 5x + 6} dx$ $= \int \frac{1}{x^2 + 5x + \frac{25}{4} - \frac{25}{4} + 6} dx$ $= \int \frac{1}{\left(x + \frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$	<p>1/2</p> <p>1/2</p>	



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1)		$= \frac{1}{2 \left(\frac{1}{2}\right)} \log \left( \frac{x + \frac{5}{2} - \frac{1}{2}}{x + \frac{5}{2} + \frac{1}{2}} \right) + c$ $= \log \left( \frac{x+2}{x+3} \right) + c$	1/2 1/2	2
	f)	Evaluate $\int_0^{\log_e 2} e^{2x} dx$		
	Ans.	$\int_0^{\log_e 2} e^{2x} dx = \left[ \frac{e^{2x}}{2} \right]_0^{\log 2}$ $= \frac{e^{2 \log 2}}{2} - \frac{e^0}{2}$ $= \frac{4}{2} - \frac{1}{2}$ $= \frac{3}{2} \text{ or } 1.5$	1/2 1/2	2
	g)	Find the area between the line $y = 2x$ and $x = 1$ and $x = 3$ .		
	Ans.	$\int_1^3 y \cdot dx = \int_1^3 2x \cdot dx$ $= \left[ 2 \cdot \frac{x^2}{2} \right]_1^3 \quad \text{or} \quad [x^2]_1^3$ $= 3^2 - 1$ $= 8$	1/2 1/2 1/2	2
	h)	Find the order and degree of the following equation: $\frac{d^2 y}{dx^2} + \sqrt{1 + \frac{dy}{dx}} = 0$		
	Ans.	Order = 2  For degree, $\left( \frac{d^2 y}{dx^2} \right)^2 = 1 + \frac{dy}{dx}$ $\therefore$ Degree = 2	1  1	2





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1)	l)	A die is thrown, find the probability of getting an odd number.		
	Ans.	$\therefore n = n(S) = 6$ $\therefore m = n(A) = 3$ $\therefore p = \frac{n(A)}{n(S)}$ $= \frac{3}{6}$ $= \frac{1}{2} \text{ or } 0.5$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2
2)		<b>Attempt any four of the following:</b>		
	a)	Find the equation of tangent and normal to the curve $y = x(2 - x)$ at $(2, 0)$ .		
	Ans.	$y = x(2 - x)$ $\therefore \frac{dy}{dx} = 2 - 2x$ $\therefore$ the slope of tangent at $(2, 0)$ is $m = 2 - 4 = -2$ $\therefore$ the equation of tangent is $y - 0 = -2(x - 2)$ $\therefore y = -2x + 4 \text{ or } 2x + y = 4$ $\therefore$ the slope of normal $= -\frac{1}{m} = \frac{1}{2}$ $\therefore$ the equation of normal is $y - 0 = \frac{1}{2}(x - 2)$ $\therefore 2y = x - 2 \text{ or } x - 2y - 2 = 0 \text{ or } -x + 2y + 2 = 0$	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	4
	b)	Find radius of curvature of the curve $x = a \cos^3 \theta$ , $y = a \sin^3 \theta$ at		
	Ans.	$\theta = \frac{\pi}{4}$ $x = a \cos^3 \theta$ $\therefore \frac{dx}{d\theta} = -3a \cos^2 \theta \sin \theta$	$\frac{1}{2}$	



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2)		$y = a \sin^3 \theta$ $\therefore \frac{dy}{d\theta} = 3a \sin^2 \theta \cos \theta$ $\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a \sin^2 \theta \cos \theta}{-3a \cos^2 \theta \sin \theta} = -\tan \theta$ $\& \frac{d^2y}{dx^2} = \frac{d}{d\theta} \left( \frac{dy}{dx} \right) \times \frac{d\theta}{dx}$ $= -\sec^2 \theta \times \frac{1}{-3a \cos^2 \theta \sin \theta}$ $\therefore \text{at } \theta = \frac{\pi}{4},$ $\frac{dy}{dx} = -\tan \frac{\pi}{4} = -1$ $\text{and } \frac{d^2y}{dx^2} = \sec^2 \frac{\pi}{4} \times \frac{1}{3a \cos^2 \frac{\pi}{4} \sin \frac{\pi}{4}}$ $= \frac{4\sqrt{2}}{3a}$ $\therefore \rho = \frac{\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[ 1 + (-1)^2 \right]^{\frac{3}{2}}}{\frac{4\sqrt{2}}{3a}}$ $= \frac{3a}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	<b>4</b>
	c) Ans.	Find the maximum and minimum value of $y = x^3 - \frac{15}{2}x^2 + 18x$ . $y = x^3 - \frac{15}{2}x^2 + 18x$ $\therefore \frac{dy}{dx} = 3x^2 - 15x + 18$ $\therefore \frac{d^2y}{dx^2} = 6x - 15$ <p>For stationary values, <math>\frac{dy}{dx} = 0</math></p> $\therefore 3x^2 - 15x + 18 = 0$ $\therefore x = 2, 3$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$	



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2)		<p>At <math>x = 2</math>, <math>\frac{d^2y}{dx^2} = 6(2) - 15 = -3 &lt; 0</math>  <math>\therefore</math> At <math>x = 2</math>, <math>y</math> has maximum value and it is  <math>y = (2)^3 - \frac{15}{2}(2)^2 + 18(2) = 14</math>            At <math>x = 3</math>, <math>\frac{d^2y}{dx^2} = 6(3) - 15 = 3 &gt; 0</math>  <math>\therefore</math> At <math>x = 3</math>, <math>y</math> has minimum value and it is  <math>y = (3)^3 - \frac{15}{2}(3)^2 + 18(3) = 13.5</math></p>	1/2 1/2 1/2 1/2	4
	d)	<p>Evaluate <math>\int \frac{e^x(x+1)}{\cos^2(xe^x)} dx</math></p>		
	Ans.	<p><math>\int \frac{e^x(x+1)}{\cos^2(xe^x)} dx</math>      <u>Put <math>xe^x = t</math></u>  <math>\therefore (xe^x + e^x \cdot 1) dx = dt</math>  <math>\therefore e^x(x+1) dx = dt</math></p> <p><math>= \int \frac{dt}{\cos^2 t}</math>  <math>= \int \sec^2 t dt</math>  <math>= \tan t + c</math>  <math>= \tan(xe^x) + c</math></p>	1 1 1/2 1 1/2	4
	e)	<p>Evaluate <math>\int \frac{\sec^2 x}{3 \tan^2 x - 2 \tan x - 5} dx</math></p>		
	Ans.	<p><math>\int \frac{\sec^2 x}{3 \tan^2 x - 2 \tan x - 5} dx</math>      <u>Put <math>\tan x = t</math></u>  <math>\therefore \sec^2 x dx = dt</math></p> <p><math>= \int \frac{1}{3t^2 - 2t - 5} dt</math>  <math>\therefore 3t^2 - 2t - 5 = 3 \left[ t^2 - \frac{2}{3}t - \frac{5}{3} \right]</math>  <math>= 3 \left[ t^2 - \frac{2}{3}t + \frac{1}{9} - \frac{1}{9} - \frac{5}{3} \right]</math>  <math>= 3 \left[ \left( t - \frac{1}{3} \right)^2 - \left( \frac{4}{3} \right)^2 \right]</math></p>	1/2 1	





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2)		$\therefore I = \int \frac{1}{3 \left[ \left( t - \frac{1}{3} \right)^2 - \left( \frac{4}{3} \right)^2 \right]} dt$ $= \frac{1}{3} \int \frac{1}{\left( t - \frac{1}{3} \right)^2 - \left( \frac{4}{3} \right)^2} dt$ $= \frac{1}{3} \cdot \frac{1}{2 \left( \frac{4}{3} \right)} \cdot \log \left[ \frac{t - \frac{1}{3} - \frac{4}{3}}{t - \frac{1}{3} + \frac{4}{3}} \right] + c$ $= \frac{1}{8} \cdot \log \left[ \frac{t - \frac{5}{3}}{t + 1} \right] + c$ $= \frac{1}{8} \cdot \log \left[ \frac{3t - 5}{3t + 3} \right] + c$ $= \frac{1}{8} \cdot \log \left[ \frac{3 \tan x - 5}{3 \tan x + 3} \right] + c$	<p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p>	4
	f)	Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$		
	Ans.	$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$ <div style="display: flex; align-items: center; margin-left: 20px;"> <div style="border-left: 1px solid black; padding-left: 10px; margin-left: 10px;"> <p>Put <math>\sin^{-1} x = t</math></p> <p><math>\therefore \frac{1}{\sqrt{1-x^2}} dx = dt</math></p> <p>Also <math>x = \sin t</math></p> </div> </div> $= \int \sin t \cdot t \cdot dt$ $= t \int \sin t dt - \int \left( \int \sin t dt \right) \frac{d}{dt}(t) dt$ $= t(-\cos t) - \int (-\cos t) \cdot 1 \cdot dt$ $= -t \cos t + \int \cos t \cdot dt$ $= -t \cos t + \sin t + c$ $= -\sin^{-1} x \cdot \cos(\sin^{-1} x) + x + c$	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p>	4



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3)		<b>Attempt any four of the following:</b>		
	a)	Evaluate $\int_0^{\pi/2} \frac{dx}{\sqrt{9-4x^2}}$		
	Ans.	The given problem cannot be solved within the given limits because for the integrating the given function within the prescribed limits the function must be well defined on the given interval. For example at $x = \frac{\pi}{2}$ , $\frac{1}{\sqrt{9-4x^2}}$ is a non-real number and hence the function is not defined on the interval $[0, \frac{\pi}{4}]$ .	4	4
	b)	Evaluate $\int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x + \cos x} dx$		
	Ans.	$I = \int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x + \cos x} dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;">                     Re place <math>x \rightarrow \pi/2 - x</math>  <math>\therefore \sin x \rightarrow \cos x</math>  <math>\&amp; \cos x \rightarrow \sin x</math> </div> $\therefore I = \int_{\pi/6}^{\pi/3} \frac{\cos x}{\cos x + \sin x} dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sin x + \cos x} dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} 1 \cdot dx$ $\therefore 2I = [x]_{\pi/6}^{\pi/3}$ $= \frac{\pi}{3} - \frac{\pi}{6}$ $\therefore I = \frac{\pi}{12}$ <p style="text-align: center;"><b>OR</b></p> $I = \int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x + \cos x} dx$ $= \int_{\pi/6}^{\pi/3} \frac{\sin(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)} dx$ $\therefore I = \int_{\pi/6}^{\pi/3} \frac{\cos x}{\cos x + \sin x} dx$	1  1  1/2  1/2  1/2  1/2	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$\therefore 2I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sin x + \cos x} dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} 1 \cdot dx$ $\therefore 2I = [x]_{\pi/6}^{\pi/3}$ $= \frac{\pi}{3} - \frac{\pi}{6}$ $\therefore I = \frac{\pi}{12}$	1 1/2 1/2 1/2 1/2	4
	c)	Find the area bounded by two curves $y^2 = x$ and $x^2 = y$		
	Ans.	<p>Given <math>y^2 = x</math> and <math>x^2 = y</math></p> $\therefore (x^2)^2 = x$ $\therefore x = 0, \quad x = 1$ $A = \int_a^b (y_2 - y_1) dx$ $= \int_0^1 [\sqrt{x} - x^2] dx$ $= \left[ \frac{2}{3} x^{3/2} - \frac{x^3}{3} \right]_0^1$ $= \left[ \frac{2}{3} \cdot 1^{3/2} - \frac{1^3}{3} \right] - 0$ $= \frac{1}{3} \quad \text{or} \quad 0.333$	1/2 + 1/2 1 1 1/2 1/2	4
		OR		
		$\therefore x = 0, \quad x = 1$ $A = \int_a^b (y_2 - y_1) dx$ $= \int_0^1 [x^2 - \sqrt{x}] dx$ $= \left[ \frac{x^3}{3} - \frac{2}{3} x^{3/2} \right]_0^1$ $= \left[ \frac{1^3}{3} - \frac{2}{3} \cdot 1^{3/2} \right] - 0$ $= -\frac{1}{3} \quad \text{or} \quad -0.333$	1/2 + 1/2 1 1 1/2	4
		$\therefore \text{the area} = \frac{1}{3} \quad \text{or} \quad 0.333$	1/2	



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3)	d)	Solve $xy^2dy - (x^3 + y^3)dx = 0$ given $y = 0$ when $x = 1$ .		
	Ans.	$xy^2dy - (x^3 + y^3)dx = 0$ $\therefore \frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}$ Put $y = vx$ $\therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$ $\therefore v + x \frac{dv}{dx} = \frac{x^3 + (vx)^3}{xv^2x^2} = \frac{1+v^3}{v^2}$ $\therefore x \frac{dv}{dx} = \frac{1+v^3}{v^2} - v$ $\therefore x \frac{dv}{dx} = \frac{1}{v^2}$ $\therefore \int v^2 dv = \int \frac{1}{x} dx$ $\therefore \frac{v^3}{3} = \log x + c$ $\therefore \frac{y^3}{3x^3} = \log x + c$ At $x=1$ & $y=0$ , $c=0$ $\therefore \frac{y^3}{3x^3} = \log x$	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	4
	e)	Solve the differential equation $(x+y)^2 \frac{dy}{dx} = a^2$		
	Ans.	Put $x+y = v$ $\therefore 1 + \frac{dy}{dx} = \frac{dv}{dx}$ or $\frac{dy}{dx} = \frac{dv}{dx} - 1$ $\therefore v^2 \left( \frac{dv}{dx} - 1 \right) = a^2$ $\therefore \frac{dv}{dx} - 1 = \frac{a^2}{v^2}$ $\therefore \frac{dv}{dx} = \frac{a^2}{v^2} + 1 = \frac{a^2 + v^2}{v^2}$ $\therefore \left( \frac{v^2}{a^2 + v^2} \right) dv = dx$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	



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3)		$\therefore \int \left( \frac{v^2}{a^2 + v^2} \right) dv = \int dx$ $\therefore \int \left( 1 - \frac{a^2}{a^2 + v^2} \right) dv = \int dx$ $\therefore v - a^2 \cdot \frac{1}{a} \tan^{-1} \left( \frac{v}{a} \right) = x + c$ $\therefore x + y - a \tan^{-1} \left( \frac{x+y}{a} \right) = x + c \quad \text{or} \quad y - a \tan^{-1} \left( \frac{x+y}{a} \right) = c$	<p>1/2</p> <p>1</p> <p>1/2</p>	4
	f)	Solve $x \frac{dy}{dx} - y = x^2$		
	Ans.	$x \frac{dy}{dx} - y = x^2$ $\therefore \frac{dy}{dx} - \frac{1}{x} \cdot y = x$ $\therefore P = -\frac{1}{x} \quad \text{and} \quad Q = x$ $\therefore IF = e^{\int p dx}$ $= e^{\int -\frac{1}{x} dx}$ $= e^{-\log x}$ $= \frac{1}{x}$ $\therefore y \cdot IF = \int Q \cdot IF \cdot dx + c$ $\therefore y \cdot \frac{1}{x} = \int x \cdot \frac{1}{x} \cdot dx + c$ $\therefore \frac{y}{x} = \int 1 \cdot dx + c$ $\therefore \frac{y}{x} = x + c$	<p>1</p> <p>1</p> <p>1</p> <p>1</p>	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		<b>Attempt any Four of the following:</b>		
	a)	Evaluate $\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+3}} dx$		
	Ans.	$I = \int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+3}} dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;">                     Re place <math>x \rightarrow 6-x</math>  <math>\therefore 9-x \rightarrow x+3</math>  <math>\&amp; x+3 \rightarrow 9-x</math> </div> $\therefore I = \int_1^5 \frac{\sqrt[3]{x+3}}{\sqrt[3]{x+3} + \sqrt[3]{9-x}} dx$ $\therefore 2I = \int_1^5 \frac{\sqrt[3]{9-x} + \sqrt[3]{x+3}}{\sqrt[3]{9-x} + \sqrt[3]{x+3}} dx$ $\therefore 2I = \int_1^5 1 \cdot dx$ $\therefore 2I = [x]_1^5$ $\therefore 2I = 5-1$ $\therefore I = 2$	1	
		<b>OR</b>		
		$I = \int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+3}} dx$ $\therefore I = \int_1^5 \frac{\sqrt[3]{9-(6-x)}}{\sqrt[3]{9-(6-x)} + \sqrt[3]{(6-x)+3}} dx$ $\therefore I = \int_1^5 \frac{\sqrt[3]{x+3}}{\sqrt[3]{x+3} + \sqrt[3]{9-x}} dx$ $\therefore 2I = \int_1^5 \frac{\sqrt[3]{9-x} + \sqrt[3]{x+3}}{\sqrt[3]{9-x} + \sqrt[3]{x+3}} dx$ $\therefore 2I = \int_1^5 1 \cdot dx$ $\therefore 2I = [x]_1^5$ $\therefore 2I = 5-1$ $\therefore I = 2$	1/2	
			1/2	
			1	
			1/2	
			1	
			1/2	
			1	
			1/2	
			1/2	<b>4</b>
			1/2	<b>4</b>



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4)	b)	Evaluate $\int \frac{dx}{4\cos^2x + 9\sin^2x}$		
	Ans.	$\int \frac{dx}{4\cos^2x + 9\sin^2x}$ $= \int \frac{dx/\cos^2x}{4\cos^2x + 9\sin^2x}$ $= \int \frac{\sec^2 x dx}{4 + 9\tan^2x}$ <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"><math>= \int \frac{dt}{4 + 9t^2}</math></div> <div style="border: 1px solid black; padding: 5px;"> <math>Put \tan x = t</math>  <math>\therefore \sec^2 x dx = dt</math> </div> </div> $= \int \frac{dt}{9\left(\frac{4}{9} + t^2\right)}$ $= \frac{1}{9} \int \frac{dt}{\left(\frac{2}{3}\right)^2 + t^2}$ $= \frac{1}{9} \cdot \frac{1}{2/3} \tan^{-1}\left(\frac{t}{2/3}\right) + c$ $= \frac{1}{6} \tan^{-1}\left(\frac{3\tan x}{2}\right) + c$	1/2 1/2 1 1 1	4
	c)	Using integration find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$		
	Ans.	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\therefore y^2 = b^2 \left(1 - \frac{x^2}{a^2}\right)$ $\therefore y^2 = \frac{b^2}{a^2} (a^2 - x^2)$ $\therefore y = \frac{b}{a} \sqrt{a^2 - x^2}$ <p>Now <math>y = 0</math> gives <math>a^2 - x^2 = 0</math> i.e., <math>x = a, -a</math></p> $\therefore A = 4 \int_0^a y dx$ $= 4 \int_0^a \frac{b}{a} \sqrt{a^2 - x^2} dx$	1 1	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$= 4 \cdot \frac{b}{a} \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) \right]_0^a$ $= \frac{4b}{a} \left[ 0 + \frac{a^2}{2} \sin^{-1}(1) \right] - 0$ $= \frac{4b}{a} \left[ \frac{a^2}{2} \cdot \frac{\pi}{2} \right]$ $= \pi ab$	1 1/2 1/2	4
	d)	<p>Solve <math>(2x^2 + 6xy - y^2)dx + (3x^2 - 2xy + y^2)dy = 0</math></p> <p>Ans. <math>M = 2x^2 + 6xy - y^2</math>  <math>\therefore \frac{\partial M}{\partial y} = 6x - 2y</math>  <math>N = 3x^2 - 2xy + y^2</math>  <math>\therefore \frac{\partial N}{\partial x} = 6x - 2y</math>  <math>\therefore</math> the equation is exact.</p> <p><math>\int_{y \text{ constant}} Mdx + \int_{\text{terms free from } x} Ndy = c</math></p> <p><math>\int (2x^2 + 6xy - y^2)dx + \int y^2 dy = c</math></p> <p><math>\therefore 2 \cdot \frac{x^3}{3} + 6y \cdot \frac{x^2}{2} - y^2 x + \frac{y^3}{3} = c</math></p> <p>or <math>\frac{2}{3}x^3 + 3x^2 y - y^2 x + \frac{y^3}{3} = c</math></p> <p style="text-align: center;"><b>OR</b></p> <p><math>(2x^2 + 6xy - y^2)dx + (3x^2 - 2xy + y^2)dy = 0</math></p> <p><math>\therefore \frac{dy}{dx} = -\frac{2x^2 + 6xy - y^2}{3x^2 - 2xy + y^2}</math></p> <p>Put <math>y = vx \quad \therefore \frac{dy}{dx} = v + x \frac{dv}{dx}</math></p> <p><math>\therefore v + x \frac{dv}{dx} = -\frac{2x^2 + 6vx^2 - v^2 x^2}{3x^2 - 2vx^2 + v^2 x^2}</math></p>	1 1/2 1/2 1 1 1 1 1/2	4





Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\therefore v + x \frac{dv}{dx} = -\frac{2+6v-v^2}{3-2v+v^2}$ $\therefore x \frac{dv}{dx} = -\frac{2+6v-v^2}{3-2v+v^2} - v$ $\therefore x \frac{dv}{dx} = \frac{-(2+6v-v^2) - v(3-2v+v^2)}{3-2v+v^2}$ $\therefore x \frac{dv}{dx} = \frac{-2-9v+3v^2-v^3}{3-2v+v^2}$ $\therefore \left[ \frac{3-2v+v^2}{-2-9v+3v^2-v^3} \right] dv = \frac{dx}{x}$ $\therefore \int \left[ \frac{3-2v+v^2}{-2-9v+3v^2-v^3} \right] dv = \int \frac{dx}{x}$ $\therefore -\frac{1}{3} \int \left[ \frac{-9+6v-3v^2}{-2-9v+3v^2-v^3} \right] dv = \int \frac{dx}{x}$ $\therefore -\frac{1}{3} \log(-2-9v+3v^2-v^3) = \log x + c$ $\therefore -\frac{1}{3} \log \left[ -2-9 \cdot \frac{y}{x} + 3 \left( \frac{y}{x} \right)^2 - \left( \frac{y}{x} \right)^3 \right] = \log x + c$	<p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p>	4
	e)	Solve $(1+x^2)dy - x^2ydx = 0$		
	Ans.	$(1+x^2)dy - x^2ydx = 0$ $\therefore \frac{1}{y} dy - \frac{x^2}{1+x^2} dx = 0$ $\therefore \int \frac{1}{y} dy - \int \frac{x^2}{1+x^2} dx = c$ $\therefore \int \frac{1}{y} dy - \int \left( 1 - \frac{1}{1+x^2} \right) dx = c$ $\therefore \log y - (x - \tan^{-1} x) = c$ <p>or <math>\log y - x + \tan^{-1} x = c</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	f)	Show that $y = \sin(\log x)$ is solution of differential equation $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$		
	Ans.	$y = \sin(\log x)$ $\therefore \frac{dy}{dx} = \cos(\log x) \cdot \frac{1}{x}$ $\therefore x \frac{dy}{dx} = \cos(\log x)$ $\therefore x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = -\sin(\log x) \cdot \frac{1}{x}$ $\therefore x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = -y$ $\therefore x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$	1 1 1 1	4
5)		<b>Attempt any Four of the following:</b>		
	a)	The probability that A can shoot at a target is $\frac{5}{7}$ and B can shoot at the same target is $\frac{3}{5}$ . (A and B shoot independently.) Find the probability that i) The target is not shot at all. ii) The target is shot by at least one of them.		
	Ans.	$P(A) = \frac{5}{7} \quad \therefore P(A') = 1 - \frac{5}{7} = \frac{2}{7}$ $P(B) = \frac{3}{5} \quad \therefore P(B') = 1 - \frac{3}{5} = \frac{2}{5}$ i) $P(\text{target is not shot}) = P(A' \& B')$ $= P(A') \cdot P(B')$ $= \frac{2}{7} \cdot \frac{2}{5}$ $= \frac{4}{35} \quad \text{or} \quad 0.114$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$ii) P(\text{at least 1 shoot}) = 1 - p(\text{target is not shot})$ $= 1 - \frac{4}{35} \quad \text{or} \quad 1 - 0.114$ $= \frac{31}{35} \quad \text{or} \quad 0.886$	1 1/2	4
		<p><b>Note for Numerical Problems:</b> For practical purpose, generally the values of fractional numbers are truncated up to 3 decimal points by the method of rounded-off. Thus the solution is taken up to 3 decimal points only. If answer is truncated more than 3 decimal points, the final answer may vary for last decimal points. Thus 31/35 is actually 0.885714285 but can be taken as 0.886. Due to the use of advance calculators, such as modern scientific non-programmable calculators, the step 31/35 may not written by the students and then directly the answer 0.012 is written. In this case, no marks to be deducted.</p>		
	b)	<p>If 30% of the bulbs produced are defective, find the probability that out of 4 bulbs selected,</p> <p>a) one is defective, b) at the most two are defective.</p>		
	Ans.	$p = \frac{30}{100} = 0.3$ $\therefore q = 0.7$ <p>Here <math>n = 4</math></p> <p>i) <math>p = {}^n C_r p^r q^{n-r}</math></p> $= {}^4 C_1 (0.3)^1 (0.7)^3$ $= 0.412$ <p>ii) <math>p = p(0) + p(1) + p(2)</math></p> $= {}^4 C_0 (0.3)^0 (0.7)^4 + {}^4 C_1 (0.3)^1 (0.7)^3 + {}^4 C_2 (0.3)^2 (0.7)^2$ $= 0.24 + 0.412 + 0.265$ $= 0.916$ <p style="text-align: center;">OR</p> $p = 1 - [p(3) + p(4)]$ $= 1 - [{}^4 C_3 (0.3)^3 (0.7)^1 + {}^4 C_4 (0.3)^4 (0.7)^0]$ $= 1 - [0.076 + 0.008]$ $= 0.916$	1 1 1 1 1	4 4





Que. No.	Sub. Que.	Model Answers	Marks	Total Marks					
5)	d)	Evaluate $\int_0^{\pi} \frac{dx}{5+4\cos x}$							
	Ans.	<p>Put <math>\tan \frac{x}{2} = t</math></p> <p><math>\therefore dx = \frac{2dt}{1+t^2}</math> and <math>\cos x = \frac{1-t^2}{1+t^2}</math></p> <table border="1" style="margin: 10px auto;"> <tr> <td>x</td> <td>0</td> <td><math>\pi</math></td> </tr> <tr> <td>t</td> <td>0</td> <td><math>\infty</math></td> </tr> </table> <p><math>\therefore \int_0^{\pi} \frac{dx}{5+4\cos x} = \int_0^{\infty} \frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}</math></p> <p><math>= 2 \int_0^{\infty} \frac{1}{t^2+9} dt</math></p> <p><math>= 2 \int_0^{\infty} \frac{1}{t^2+3^2} dt</math></p> <p><math>= 2 \times \frac{1}{3} \left[ \tan^{-1} \left( \frac{t}{3} \right) \right]_0^{\infty}</math></p> <p><math>= \frac{2}{3} \left[ \tan^{-1} \infty - \tan^{-1} 0 \right]</math></p> <p><math>= \frac{2}{3} \left[ \frac{\pi}{2} \right]</math></p> <p><math>= \frac{\pi}{3}</math></p> <hr style="border-top: 1px dashed black;"/>	x	0	$\pi$	t	0	$\infty$	1/2 1/2 1/2 1/2 1 1/2 1/2
x	0	$\pi$							
t	0	$\infty$							
	e)	Evaluate $\int \frac{x}{x^2+3x-4} dx$							
	Ans.	<p><math>\int \frac{x}{x^2+3x-4} dx</math></p> <p><math>= \int \frac{x}{(x-1)(x+4)} dx</math></p> <p><math>= \int \left[ \frac{1/5}{x-1} + \frac{4/5}{x+4} \right] dx</math> (Please refer next note)</p> <p><math>= \frac{1}{5} \log(x-1) + \frac{4}{5} \log(x+4) + c</math></p>	2 1+1	4					





Que. No.	Sub. Que.	Model Answers	Marks	Total Marks														
6)		<p>i) <math>P(A' \cap B') = P(A \cup B)</math>  <math>= 1 - P(A \cup B)</math>  <math>= 1 - \frac{2}{3}</math>  <math>= \frac{1}{3}</math> or 0.333</p> <p>ii) <math>P(A \cap B) = P(A) + P(B) - P(A \cup B)</math>  <math>= \frac{1}{2} + \frac{1}{3} - \frac{2}{3}</math>  <math>= \frac{1}{6}</math> or 0.167</p> <p><math>\therefore P(A/B) = \frac{P(A \cap B)}{P(B)}</math>  <math>= \frac{1/6}{1/3}</math>  <math>= \frac{1}{2}</math> or 0.5</p>	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p>	4														
	b)	<p>If the probability that an electric motor is defective is 0.01, what is the probability that the sample of 300 electric motors will contain exactly 5 defective motors? (<math>e^{-3} = 0.0498</math>)</p>																
	Ans.	<p><math>p = 0.01</math>                      <math>n = 300</math>  <math>\therefore m = np = 0.01 \times 300 = 3</math>  <math>p(r) = \frac{e^{-m} \cdot m^r}{r!}</math>  <math>\therefore p(5) = \frac{e^{-3} \cdot 3^5}{5!}</math>  <math>= 0.101</math></p>	<p>1</p> <p>1 1/2</p> <p>1 1/2</p>	4														
	c)	<p>Fit a Poisson distribution for the following observation:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>20</td> <td>30</td> <td>40</td> <td>50</td> <td>60</td> <td>70</td> </tr> <tr> <td>f</td> <td>8</td> <td>12</td> <td>30</td> <td>10</td> <td>6</td> <td>4</td> </tr> </table>	x	20	30	40	50	60	70	f	8	12	30	10	6	4		
x	20	30	40	50	60	70												
f	8	12	30	10	6	4												



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks																								
6)	Ans.	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>f</th> <th>xy</th> </tr> </thead> <tbody> <tr> <td>20</td> <td>8</td> <td>160</td> </tr> <tr> <td>30</td> <td>12</td> <td>360</td> </tr> <tr> <td>40</td> <td>30</td> <td>1200</td> </tr> <tr> <td>50</td> <td>10</td> <td>500</td> </tr> <tr> <td>60</td> <td>6</td> <td>360</td> </tr> <tr> <td>70</td> <td>4</td> <td>280</td> </tr> <tr> <td></td> <td><b>70</b></td> <td><b>2860</b></td> </tr> </tbody> </table> <p> <math>\therefore \text{mean } m = \frac{2860}{70} = 40.857</math>  <math>\therefore p = \frac{e^{-m} m^r}{r!}</math>  <math>= \frac{e^{-40.857} (40.857)^r}{r!}</math> </p> <p>-----</p>	x	f	xy	20	8	160	30	12	360	40	30	1200	50	10	500	60	6	360	70	4	280		<b>70</b>	<b>2860</b>	1  2  1	4
x	f	xy																										
20	8	160																										
30	12	360																										
40	30	1200																										
50	10	500																										
60	6	360																										
70	4	280																										
	<b>70</b>	<b>2860</b>																										
	d)	A metal wire 36 m long is bent to form a rectangle. Find its dimensions when its area is maximum.																										
	Ans.	<p>Let x and y be the sides of rectangle.</p> <p><math>\therefore 2x + 2y = 36</math> or <math>x + y = 18</math>  <math>\therefore y = 18 - x</math>  <i>But area</i> <math>A = xy = x(18 - x) = 18x - x^2</math>  <math>\therefore \frac{dA}{dx} = 18 - 2x</math>  <math>\therefore \frac{d^2A}{dx^2} = -2</math>  <i>For stationary values,</i> <math>\frac{dA}{dx} = 0</math>  <math>\therefore 18 - 2x = 0</math>  <math>\therefore x = 9</math>  <i>At</i> <math>x = 9</math>, <math>\frac{d^2A}{dx^2} = -2 &lt; 0</math>  <math>\therefore</math> <i>At</i> <math>x = 9</math>, <i>A has maximum value</i>  <i>and the other side is</i>  <math>y = 18 - x = 9</math> </p>	1/2  1  1/2  1/2	4																								
			1/2																									







Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	f)	Find the area between the parabola $y = 4 - x^2$ and the x-axis.		
	Ans.	$y = 4 - x^2$ and $x$ -axis i.e., $y = 0$ $\therefore 4 - x^2 = 0$ $\therefore x = -2, 2$ $\therefore A = \int_a^b y dx$ $= \int_{-2}^2 (4 - x^2) dx$ $= \left[ 4x - \frac{x^3}{3} \right]_{-2}^2$ $= \left[ 2^3 - \frac{2^3}{3} \right] - \left[ (-2)^3 - \frac{(-2)^3}{3} \right]$ $= \frac{32}{3}$ or 10.667	$\frac{1}{2} + \frac{1}{2}$  $\frac{1}{2}$  1  $\frac{1}{2}$  1	4
		<p style="text-align: center;"><b>Important Note</b></p> <p>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. <b>In such case, FIRST SEE whether the method falls within the scope of the curriculum, and THEN ONLY give appropriate marks in accordance with the scheme of marking.</b></p>		