



Winter 2014 Examination

Subject & Code: Basic Maths (17104)

Model Answer

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Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		<p><b>Important Instructions to the Examiners:</b></p> <ol style="list-style-type: none"><li>1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme.</li><li>2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.</li><li>3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)</li><li>4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.</li><li>5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer.</li><li>6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.</li><li>7) For programming language papers, credit may be given to any other program based on equivalent concept.</li></ol>		



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1)		<b>Attempt any TEN of the following:</b>		
	a)	Find the value of $\begin{vmatrix} 2 & 3 & 5 \\ 1 & 4 & 2 \\ 3 & 1 & 6 \end{vmatrix}$		
	Ans.	$\begin{vmatrix} 2 & 3 & 5 \\ 1 & 4 & 2 \\ 3 & 1 & 6 \end{vmatrix} = 2(24-2) - 3(6-6) + 5(1-12)$ $= -11$	1 1	2
	b)	If $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ , find the matrix B such that $2A + 3B = 0$		
	Ans.	$2A = 2 \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ 4 & 8 \end{bmatrix}$ $\therefore 3B = -2A = \begin{bmatrix} -6 & 2 \\ -4 & -8 \end{bmatrix}$ $\therefore B = \frac{1}{3} \begin{bmatrix} -6 & 2 \\ -4 & -8 \end{bmatrix}$	1 $\frac{1}{2}$ $\frac{1}{2}$	2
		<b>OR</b>		
		$2A + 3B = 0$ $\therefore 3B = -2A$ $= -2 \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ $= \begin{bmatrix} -6 & 2 \\ -4 & -8 \end{bmatrix}$ $\therefore B = \frac{1}{3} \begin{bmatrix} -6 & 2 \\ -4 & -8 \end{bmatrix}$	$\frac{1}{2}$ 1 $\frac{1}{2}$	2
	c)	Find the value of a and b, if $\begin{bmatrix} a-4b & 5 \\ 6 & -a+b \end{bmatrix} = \begin{bmatrix} 11 & 5 \\ 6 & -5 \end{bmatrix}$		
	Ans.	$\begin{bmatrix} a-4b & 5 \\ 6 & -a+b \end{bmatrix} = \begin{bmatrix} 11 & 5 \\ 6 & -5 \end{bmatrix}$ $\therefore a-4b = 11$ $-a+b = -5$ $\hline \therefore -3b = 6$ $\therefore \boxed{b = -2}$ $\therefore \boxed{a = 3}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2





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1)		<p style="text-align: center;"><b>OR</b></p> $\frac{x}{x^2-x-2} = \frac{x}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$ $\therefore x = (x-2)A + (x+1)B$ $\therefore \frac{1}{3} = A$ $\therefore \frac{2}{3} = B$ $\therefore \frac{x}{x^2-x-2} = \frac{1}{3} + \frac{2}{3}$ <p><b>Note for partial fraction problems:</b> The problems of partial fractions could also be solved by the method of "equating equal power coefficients". This method is also applicable. Give appropriate marks in accordance with the scheme of marking in the later problems as the solution by this method is not discussed. For the sake of convenience, the solution of the above problem with the help of this method is illustrated hereunder.</p> $\frac{x}{x^2-x-2} = \frac{x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$ $\therefore x = (x+1)A + (x-2)B$ $\therefore x = xA + A + xB - 2B$ $\therefore 1 \cdot x + 0 = x(A+B) + (A-2B)$ $\therefore A+B=1$ $A-2B=0$ $\therefore 2A+2B=2$ $\underline{A-2B=0}$ $\therefore 3A=2$ $\therefore A = \frac{2}{3}$ $\therefore B = 1 - A = 1 - \frac{2}{3}$ $\therefore B = \frac{1}{3}$ $\therefore \frac{x}{x^2-x-2} = \frac{2}{3} + \frac{1}{3}$	<p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p>	<p>2</p> <p>2</p>



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	f)	Show that $\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$		
	Ans.	$\tan\left(\frac{\pi}{4}-\theta\right)=\frac{\tan\left(\frac{\pi}{4}\right)-\tan\theta}{1+\tan\left(\frac{\pi}{4}\right)\tan\theta}$ $=\frac{1-\tan\theta}{1+\tan\theta}$	1 1	2
	g)	Prove that $\cos 2A = 2\cos^2 A - 1$		
	Ans.	$\cos 2A = \cos(A+A)$ $= \cos A \cos A - \sin A \sin A$ $= \cos^2 A - \sin^2 A$ $= \cos^2 A - (1 - \cos^2 A)$ $= \cos^2 A - 1 + \cos^2 A$ $= 2\cos^2 A - 1$ <p style="text-align: center;"><b>OR</b></p> $\cos 2A = \cos^2 A - \sin^2 A$ $= \cos^2 A - (1 - \cos^2 A)$ $= 2\cos^2 A - 1$	1/2 1/2 1/2 1/2	2
	h)	If $\sin A = 0.4$ , find the value of $\sin 3A$ .		
	Ans.	$\sin 3A = 3\sin A - 4\sin^3 A$ $= 3(0.4) - 4(0.4)^3$ $= 0.944 \quad \dots(*)$ <p style="text-align: center;"><b>Note (*):</b> Due to the use of advance scientific calculator, writing directly the step (*) is allowed. No marks to be deducted.</p> <p style="text-align: center;"><b>OR</b></p> Given that $\sin A = 0.4$ . $\therefore A = \sin^{-1}(0.4) = 23.578^\circ$ $\therefore \sin 3A = \sin(3 \times 23.578^\circ)$ $= 0.944$	1 1/2 1/2	2
			1 1/2 1/2	2



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	i)	Prove that $\frac{\cos 3\theta}{\cos \theta} + \frac{\sin 3\theta}{\sin \theta} = 4 \cos 2\theta$		
	Ans.	$\frac{\cos 3\theta}{\cos \theta} + \frac{\sin 3\theta}{\sin \theta} = \frac{\sin \theta \cos 3\theta + \cos \theta \sin 3\theta}{\cos \theta \sin \theta}$ $= \frac{\sin(\theta + 3\theta)}{\cos \theta \sin \theta}$ $= \frac{\sin 4\theta}{\cos \theta \sin \theta}$ $= \frac{\sin 2(2\theta)}{\cos \theta \sin \theta}$ $= \frac{2 \sin 2\theta \cos 2\theta}{\cos \theta \sin \theta}$ $= \frac{2 \cdot 2 \sin \theta \cos \theta \cdot \cos 2\theta}{\cos \theta \sin \theta}$ $= 4 \cos 2\theta$ <p style="text-align: center;"><b>OR</b></p> $\frac{\cos 3\theta}{\cos \theta} + \frac{\sin 3\theta}{\sin \theta} = \frac{4 \cos^3 \theta - 3 \cos \theta}{\cos \theta} + \frac{3 \sin \theta - 4 \sin^3 \theta}{\sin \theta}$ $= 4 \cos^2 \theta - 3 + 3 - 4 \sin^2 \theta$ $= 4 \cos^2 \theta - 4 \sin^2 \theta$ $= 4(\cos^2 \theta - \sin^2 \theta)$ $= 4 \cos 2\theta$	1/2 1/2 1/2 1/2	2
	j)	Evaluate without using calculator $\frac{\tan 66^\circ + \tan 69^\circ}{1 - \tan 66^\circ \tan 69^\circ}$		
	Ans.	$\frac{\tan 66^\circ + \tan 69^\circ}{1 - \tan 66^\circ \tan 69^\circ} = \tan(66^\circ + 69^\circ)$ $= \tan 135^\circ$ $= \tan(90^\circ + 45^\circ) \quad \text{OR} \quad \tan(180^\circ - 45^\circ)$ $= -\cot 45^\circ \quad \text{OR} \quad -\tan(45^\circ)$ $= -1$	1/2 1/2 1/2 1/2	2



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1)	k)	Find the slope and y-intercept of the line $\frac{x}{4} - \frac{y}{3} = 2$		
	Ans.	$\frac{x}{4} - \frac{y}{3} - 2 = 0$ $\therefore a = \frac{1}{4} \qquad b = -\frac{1}{3} \qquad c = -2$ $\therefore \text{slope } m = -\frac{a}{b} = -\frac{\frac{1}{4}}{-\frac{1}{3}} = \frac{3}{4} \text{ or } 0.75$ $y\text{-int} = -\frac{c}{b} = -\frac{-2}{-\frac{1}{3}} = -6$ <p style="text-align: center;"><b>OR</b></p> $\frac{x}{4} - \frac{y}{3} = 2$ $\therefore 3x - 4y - 24 = 0$ $\therefore a = 3 \qquad b = -4 \qquad c = -24$ $\therefore \text{slope } m = -\frac{a}{b} = -\frac{3}{-4} = \frac{3}{4} \text{ or } 0.75$ $y\text{-int} = -\frac{c}{b} = -\frac{-24}{-4} = -6$ <p style="text-align: center;"><b>OR</b></p> $\frac{x}{4} - \frac{y}{3} = 2$ $\therefore y = \frac{3}{4}x - 6$ $\therefore \text{slope } m = \frac{3}{4} \text{ or } 0.75$ $y\text{-int} = -6$ <hr/>	1 1	2
	l)	Find the range of the following: 2, 3, 1, 10, 6, 31, 17, 20, 24		
Ans.	$L = 31 \quad S = 1$ $\therefore \text{Range} = L - S$ $= 31 - 1$ $= 30$		1 1	2







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2)		<p><b>Note:</b> As the use of the advance scientific calculator is permissible, calculating directly the values of fractional quantities e.g., <math>\frac{1}{4}\left(\frac{1}{12}-\frac{1}{45}\right)+\frac{1}{3}\left(\frac{1}{18}+\frac{1}{35}\right)+\frac{1}{2}\left(-\frac{1}{27}-\frac{1}{14}\right)</math> is allowed. The same is also applicable in the next alternative method. No marks to be deducted for such direct calculations.</p> <p style="text-align: center;"><b>OR</b></p> $3x-4y+6z=60$ $10x+15y-6z=330$ $18x-14y+21z=-252$ $\therefore D = \begin{vmatrix} 3 & -4 & 6 \\ 10 & 15 & -6 \\ 18 & -14 & 21 \end{vmatrix} = 3(315-84)+4(210+108)+6(-140-270)$ $= -495$ $D_y = \begin{vmatrix} 3 & 60 & 6 \\ 10 & 330 & -6 \\ 18 & -252 & 21 \end{vmatrix} = 3(6930-1512)-60(210+108)+6(-2520-5940)$ $= -53586$ $D_z = \begin{vmatrix} 3 & -4 & 60 \\ 10 & 15 & 330 \\ 18 & -14 & -252 \end{vmatrix} = 3(-3780+4620)+4(-2520-5940)+60(-140-270)$ $= -55920$ $\therefore y = \frac{D_y}{D} = \frac{-53586}{-495} = 108.255$ $z = \frac{D_z}{D} = \frac{-55920}{-495} = 112.970$ <hr style="border-top: 1px dashed black;"/> <p>b) If <math>A = \begin{bmatrix} 2 &amp; -1 &amp; 1 \\ -2 &amp; 3 &amp; -2 \\ -4 &amp; 4 &amp; -3 \end{bmatrix}</math>, find <math>A^2</math>.</p> <p>Ans. <math>A^2 = A \cdot A = \begin{bmatrix} 2 &amp; -1 &amp; 1 \\ -2 &amp; 3 &amp; -2 \\ -4 &amp; 4 &amp; -3 \end{bmatrix} \begin{bmatrix} 2 &amp; -1 &amp; 1 \\ -2 &amp; 3 &amp; -2 \\ -4 &amp; 4 &amp; -3 \end{bmatrix}</math></p> $= \begin{bmatrix} 4+2-4 & -2-3+4 & 2+2-3 \\ -4-6+8 & 2+9-8 & -2-6+6 \\ -8-8+12 & 4+12-12 & -4-8+9 \end{bmatrix}$ $= \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix} \quad \text{(Please check note on next page)}$	1  1  1  $\frac{1}{2}$  $\frac{1}{2}$	4
			2  2	4



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2)		<p><b>Note: In the answer matrix of <math>A^2</math>, if 1 to 3 elements are wrong either in sign or value, deduct <math>\frac{1}{2}</math> marks; if 4 to 6 elements are wrong, you may deduct 1 mark; other deduct all 2 marks.</b></p>		
	c)	If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ , $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ , $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ , verify that $A(B+C) = AB + AC$ .		
	Ans.	$B + C = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ $= \begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$ $\therefore A(B+C) = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$ $= \begin{bmatrix} -1+8 & 2+6 \\ 2+12 & -4+9 \end{bmatrix}$ $= \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$ $AB = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ $= \begin{bmatrix} 2+4 & 1+6 \\ -4+6 & -2+9 \end{bmatrix}$ $= \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix}$ $AC = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ $= \begin{bmatrix} -3+4 & 1+0 \\ 6+6 & -2+0 \end{bmatrix}$ $= \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix}$ $\therefore AB + AC = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix}$ $= \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$ $\therefore A(B+C) = AB + AC$	1  1  $\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$	4



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2)	d)	If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ , find $A^2 - 3A + 9I$ , where I is the unit matrix		
	Ans.	$A^2 = A \cdot A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ $= \begin{bmatrix} 1-4-9 & -2-6+3 & 3+2+6 \\ 2+6+3 & -4+9-1 & 6-3-2 \\ -3+2-6 & 6+3+2 & -9-1+4 \end{bmatrix}$ $= \begin{bmatrix} -12 & -5 & 11 \\ 11 & 4 & 1 \\ -7 & 11 & -6 \end{bmatrix}$ $3A = 3 \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -6 & 9 \\ 6 & 9 & -3 \\ -9 & 3 & 6 \end{bmatrix}$ $9I = 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ $\therefore A^2 - 3A + 9I = \begin{bmatrix} -12 & -5 & 11 \\ 11 & 4 & 1 \\ -7 & 11 & -6 \end{bmatrix} - \begin{bmatrix} 3 & -6 & 9 \\ 6 & 9 & -3 \\ -9 & 3 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ $= \begin{bmatrix} -12-3+9 & -5+6+0 & 11-9+0 \\ 11-6+0 & 4-9+9 & 1+3+0 \\ -7+9+0 & 11-3+0 & -6-6+9 \end{bmatrix}$ $= \begin{bmatrix} -6 & 1 & 2 \\ 5 & 4 & 4 \\ 2 & 8 & -3 \end{bmatrix}$	1  1  $\frac{1}{2}$  $\frac{1}{2}$       1	4
		<p><b>Note:</b> The above problem could also be solved by taking all the terms simultaneously as follows:</p> $A^2 - 3A + 9I$ $= \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix} - 3 \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix} + 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		



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2)		$= \begin{bmatrix} 1-4-9 & -2-6+3 & 3+2+6 \\ 2+6+3 & -4+9-1 & 6-3-2 \\ -3+2-6 & 6+3+2 & -9-1+4 \end{bmatrix} - \begin{bmatrix} 3 & -6 & 9 \\ 6 & 9 & -3 \\ -9 & 3 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ $= \begin{bmatrix} -12 & -5 & 11 \\ 11 & 4 & 1 \\ -7 & 11 & -6 \end{bmatrix} - \begin{bmatrix} 3 & -6 & 9 \\ 6 & 9 & -3 \\ -9 & 3 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ $= \begin{bmatrix} -12-3+9 & -5+6+0 & 11-9+0 \\ 11-6+0 & 4-9+9 & 1+3+0 \\ -7+9+0 & 11-3+0 & -6-6+9 \end{bmatrix}$ $= \begin{bmatrix} -6 & 1 & 2 \\ 5 & 4 & 4 \\ 2 & 8 & -3 \end{bmatrix}$	<p>1+1/2+</p> <p>1/2</p> <p>1</p> <p>1</p>	4
e)	<p>Resolve into partial fractions: <math>\frac{x^2+1}{2x^4+5x^2+2}</math></p> <p>Ans. <math>\frac{x^2+1}{2x^4+5x^2+2} \quad (\text{Put } x^2 = y)</math></p> $= \frac{y+1}{2y^2+5y+2}$ $= \frac{y+1}{(2y+1)(y+2)} = \frac{A}{2y+1} + \frac{B}{y+2}$ $\therefore \boxed{y+1 = (y+2)A + (2y+1)B}$ <p>Put <math>2y+1=0</math> or <math>y = -\frac{1}{2}</math></p> $\therefore -\frac{1}{2}+1 = \left(-\frac{1}{2}+2\right)A + 0$ $\therefore \frac{1}{2} = \frac{3}{2}A$ $\therefore \boxed{\frac{1}{3} = A}$ <p>Put <math>y+2=0</math> or <math>y = -2</math></p> $\therefore -2+1 = 0 + (-4+1)B$ $\therefore -1 = -3B$ $\therefore \boxed{\frac{1}{3} = B}$	<p>1</p> <p>1</p> <p>1</p>		



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2)		$\therefore \frac{y+1}{2y^2+5y+2} = \frac{\frac{1}{3}}{2y+1} + \frac{\frac{1}{3}}{y+2}$ $\therefore \frac{x^2+1}{2x^4+5x^2+2} = \frac{\frac{1}{3}}{2x^2+1} + \frac{\frac{1}{3}}{x^2+2}$	1/2 1/2	4
	f)	Resolve into partial fractions: $\frac{x^3+x}{x^2-9}$		
	Ans.	$\frac{x^3+x}{x^2-9} = x + \frac{10x}{x^2-9}$ $\therefore \frac{10x}{x^2-9} = \frac{10x}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$ $\therefore \boxed{10x = (x+3)A + (x-3)B}$ <p>Put <math>x-3=0</math> i.e., <math>x=3</math></p> $\therefore 30 = 6A + 0$ $\therefore \boxed{5 = A}$ <p>Put <math>x+3=0</math> i.e., <math>x=-3</math></p> $\therefore -30 = 0 - 6B$ $\therefore \boxed{5 = B}$ $\therefore \frac{10x}{x^2-9} = \frac{5}{x-3} + \frac{5}{x+3}$ $\therefore \boxed{\frac{x^3+x}{x^2-9} = x + \frac{5}{x-3} + \frac{5}{x+3}}$	1 1 1 1/2 1/2	4
3)	a)	Attempt any FOUR of the following: Solve the equations $x+2y+3z=1$ , $2x+3y+2z=2$ , $3x+2y+4z=1$ by using matrix inversion method.		
	Ans.	$x+2y+3z=1$ $2x+3y+2z=2$ $3x+2y+4z=1$		



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3)		$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad K = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $\therefore  A  = 1(12-4) - 2(8-6) + 3(2-9) = -11$ $C(A) = \begin{bmatrix} \begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} \end{bmatrix}$ $= \begin{bmatrix} 8 & -2 & -5 \\ -2 & -5 & 4 \\ -5 & 4 & -1 \end{bmatrix}$ <p style="text-align: center;"><b>OR</b></p> <p>The minor matrix of A is</p> $M(A) = \begin{bmatrix} \begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} \end{bmatrix}$ $= \begin{bmatrix} 8 & 2 & -5 \\ 2 & -5 & -4 \\ -5 & -4 & -1 \end{bmatrix}$ <p><math>\therefore</math> the matrix of cofactors is,</p> $\therefore C(A) = \begin{bmatrix} 8 & -2 & -5 \\ -2 & -5 & 4 \\ -5 & 4 & -1 \end{bmatrix}$ <p style="text-align: center;"><b>OR</b></p> <p>The minors of matrix A are</p> $A_{11} = \begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} = 8 \quad A_{12} = -\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = -2 \quad A_{13} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -5$ $A_{21} = -\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = -2 \quad A_{22} = \begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} = -5 \quad A_{23} = -\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 4$ $A_{31} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -5 \quad A_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = 4 \quad A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$	<p>1</p> <p>1</p> <p>1/2</p> <p>OR</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>OR</p> <p>1</p>	



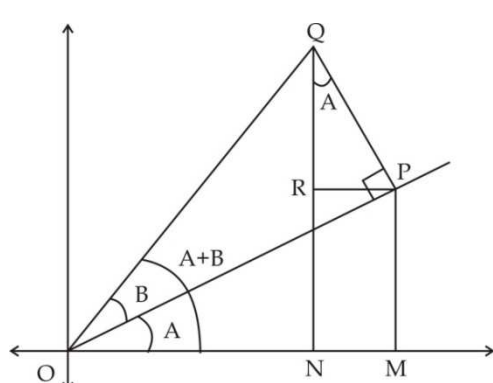
Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		<p><math>\therefore</math> the matrix of cofactors is,</p> $\therefore C(A) = \begin{bmatrix} 8 & -2 & -5 \\ -2 & -5 & 4 \\ -5 & 4 & -1 \end{bmatrix}$ $\therefore \text{adj}(A) = \begin{bmatrix} 8 & -2 & -5 \\ -2 & -5 & 4 \\ -5 & 4 & -1 \end{bmatrix}$ $\therefore A^{-1} = \frac{1}{-11} \begin{bmatrix} 8 & -2 & -5 \\ -2 & -5 & 4 \\ -5 & 4 & -1 \end{bmatrix}$ $\therefore X = A^{-1}K = \frac{1}{-11} \begin{bmatrix} 8 & -2 & -5 \\ -2 & -5 & 4 \\ -5 & 4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $= \frac{1}{-11} \begin{bmatrix} -1 \\ -8 \\ 2 \end{bmatrix}$ $= \begin{bmatrix} \frac{1}{11} \\ \frac{8}{11} \\ -\frac{2}{11} \end{bmatrix}$ <p><math>\therefore x = \frac{1}{11} \quad y = \frac{8}{11} \quad z = -\frac{2}{11}</math></p> <hr style="border-top: 1px dashed black;"/>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>	<p>4</p>
	b)	<p>Resolve into partial fractions: <math>\frac{x^2 + 23x}{(x-3)(x^2 + 1)}</math></p>		
	Ans.	$\frac{x^2 + 23x}{(x-3)(x^2 + 1)} = \frac{A}{x-3} + \frac{Bx+C}{x^2 + 1}$ $\therefore x^2 + 23x = (x-3)(x^2 + 1) \left[ \frac{A}{x-3} + \frac{Bx+C}{x^2 + 1} \right]$ $\therefore \boxed{x^2 + 23x = (x^2 + 1)A + (x-3)(Bx+C)}$		

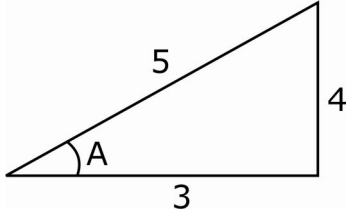






Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	c)	Resolve into partial fractions: $\frac{e^x + 1}{2e^{2x} + 7e^x + 5}$		
	Ans.	$\frac{e^x + 1}{2e^{2x} + 7e^x + 5} \quad (\text{Put } e^x = y)$ $= \frac{y+1}{2y^2 + 7y + 5}$ $= \frac{y+1}{(2y+5)(y+1)}$ $= \frac{1}{2y+5}$ $= \frac{1}{2e^x + 5}$	1	4
		<b>OR</b>		
		$\frac{e^x + 1}{2e^{2x} + 7e^x + 5} \quad (\text{Put } e^x = y)$ $= \frac{y+1}{2y^2 + 7y + 5}$ $= \frac{y+1}{(2y+5)(y+1)} = \frac{A}{2y+5} + \frac{B}{y+1}$ $\therefore \boxed{y+1 = (y+1)A + (2y+5)B}$ $\text{Put } 2y+5=0 \quad \therefore y = -\frac{5}{2}$ $\therefore -\frac{5}{2} + 1 = \left(-\frac{5}{2} + 1\right)A + 0$ $\therefore -\frac{3}{2} = -\frac{3}{2}A$ $\therefore \boxed{1 = A}$ $\text{Put } y+1=0 \quad \therefore y = -1$ $\therefore -1 + 1 = 0 + (-2 + 5)B$ $\therefore 0 = 3B$ $\therefore \boxed{0 = B}$ $\therefore \frac{y+1}{2y^2 + 7y + 5} = \frac{1}{2y+5} + \frac{0}{y+1}$ $\therefore \boxed{\frac{e^x + 1}{2e^{2x} + 7e^x + 5} = \frac{1}{2e^x + 5}}$	1	
			1	
			1/2	
			1/2	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	d)	<p>Prove that <math>\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B</math></p>	1	4
	Ans.	<div style="text-align: center;">  </div> $\begin{aligned} \sin(A+B) &= \frac{QN}{OQ} \\ &= \frac{QR+RN}{OQ} \\ &= \frac{QR+PM}{OQ} \\ &= \frac{QR}{OQ} + \frac{PM}{OQ} \\ &= \frac{QR}{PQ} \times \frac{PQ}{OQ} + \frac{PM}{OP} \times \frac{OP}{OQ} \\ &= \cos A \cdot \sin B + \sin A \cdot \cos B \end{aligned}$	1	
		<p><b>Note:</b> The above is proved by different ways in several books. Consider all these proof but check whether the method is falling within the scope of curriculum and give appropriate marks in accordance with the scheme of marking. In accordance with the Teacher's Manual published by MSBTE, the result is treated as Fundamental Result which is not proved by the help of any another result. If the above result is proved by students using any another result, suppose using <math>\cos(A+B)</math>, then this result i.e., <math>\cos(A+B)</math> must have been proved first.</p>	1	
	e)	<p>Prove that <math>2 \cot^{-1}(3) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}</math></p>	1	
	Ans.	$2 \cot^{-1}(3) = 2 \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{2 \cdot \frac{1}{3}}{1 - \left(\frac{1}{3}\right)^2}\right) = \tan^{-1}\left(\frac{3}{4}\right)$	1+1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		<p><b>OR</b></p> $2 \cot^{-1}(3) = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{3}\right)$ $= \tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{3}}{1 - \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)}\right)$ $= \tan^{-1}\left(\frac{3}{4}\right)$ <p>Let <math>A = \operatorname{cosec}^{-1}\left(\frac{5}{4}\right)</math></p> $\therefore \operatorname{cosec} A = \frac{5}{4}$ <div style="text-align: center;">  </div> $\therefore 2 \cot^{-1}(3) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{4}{3}\right)$ $= \tan^{-1}\left(\frac{\frac{3}{4} + \frac{4}{3}}{1 - \left(\frac{3}{4}\right)\left(\frac{4}{3}\right)}\right)$ $= \tan^{-1}(\infty)$ $= \frac{\pi}{2}$ <p style="text-align: center;"><b>OR</b></p> $\therefore 2 \cot^{-1}(3) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \tan^{-1}\left(\frac{3}{4}\right) + \cot^{-1}\left(\frac{3}{4}\right)$ $= \frac{\pi}{2}$	<p>OR</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>OR</p> <p>1</p> <p>1</p>	<p><b>4</b></p>
		<p><b>Note that the result <math>\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}</math> can be used directly</b></p>		



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	f)	Prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$		
	Ans.	$\begin{aligned} \tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) &= \frac{\pi}{4} + \tan^{-1}(2) + \tan^{-1}(3) \\ &= \frac{\pi}{4} + \pi + \tan^{-1}\left(\frac{2+3}{1-2 \cdot 3}\right) \\ &= \frac{\pi}{4} + \pi + \tan^{-1}(-1) \\ &= \frac{\pi}{4} + \pi - \frac{\pi}{4} \\ &= \pi \end{aligned}$ <p style="text-align: center;"><b>OR</b></p> $\begin{aligned} \tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) &= \pi + \tan^{-1}\left(\frac{1+2}{1-1 \cdot 2}\right) + \tan^{-1}(3) \\ &= \pi + \tan^{-1}(-3) + \tan^{-1}(3) \\ &= \pi - \tan^{-1}(3) + \tan^{-1}(3) \\ &= \pi \end{aligned}$ <p style="text-align: center;"><b>OR</b></p> $\begin{aligned} \tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) &= \tan^{-1}(1) + \pi + \tan^{-1}\left(\frac{2+3}{1-2 \cdot 3}\right) \\ &= \tan^{-1}(1) + \pi + \tan^{-1}(-1) \\ &= \tan^{-1}(1) + \pi - \tan^{-1}(1) \\ &= \pi \end{aligned}$	1 1 1 ½ ½	4
				1 1 1 1
4)	a)	<p>Without using the calculator, find the value of</p> $\frac{4}{3 \tan^2 30^\circ} + 3 \sin^2 120^\circ - \operatorname{cosec}^2 30^\circ - \frac{3}{4 \cot^2 120^\circ} + \cos^2 270^\circ$		
	Ans.	$\tan^2 30^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$ $\sin 120^\circ = \sin(90^\circ + 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$	½ ½	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\therefore \sin^2 120^\circ = \frac{3}{4}$ $\cos ec 30^\circ = 2$ $\therefore \cos ec^2 30^\circ = 4$ $\cot 120^\circ = \cot(90^\circ + 30^\circ)$ $= -\tan 30^\circ$ $= -\frac{1}{\sqrt{3}}$ $\therefore \cot^2 120^\circ = \frac{1}{3}$ $\cos 270^\circ = \cos(3 \times 90^\circ + 0)$ $= \sin 0$ $= 0$ $\therefore \cos^2 270^\circ = 0$ <p><i>But given that</i></p> $\frac{4}{3 \tan^2 30^\circ} + 3 \sin^2 120^\circ - \cos ec^2 30^\circ - \frac{3}{4 \cot^2 120^\circ} + \cos^2 270^\circ$ $= \frac{4}{3 \left(\frac{1}{3}\right)} + 3 \left(\frac{3}{4}\right) - 4 - \frac{3}{4 \left(\frac{1}{3}\right)} + 0$ $= \frac{9}{2} \text{ or } 4.5$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	4
	b)	<p>Prove that <math>\frac{\cos 3A + 2 \cos 5A + \cos 7A}{\cos A + 2 \cos 3A + \cos 5A} = \cos 2A - \sin 2A \tan 3A</math></p>		
	Ans.	$\frac{\cos 3A + 2 \cos 5A + \cos 7A}{\cos A + 2 \cos 3A + \cos 5A} = \frac{\cos 3A + \cos 7A + 2 \cos 5A}{\cos A + \cos 5A + 2 \cos 3A}$ $= \frac{2 \cos 5A \cos(-2A) + 2 \cos 5A}{2 \cos 3A \cos(-2A) + 2 \cos 3A}$ $= \frac{\cos 5A [2 \cos(-2A) + 2]}{\cos 3A [2 \cos(-2A) + 2]}$ $= \frac{\cos 5A}{\cos 3A}$ $= \frac{\cos(2A + 3A)}{\cos 3A}$	<p>1</p> <p>1</p> <p>1/2</p>	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$= \frac{\cos 2A \cos 3A - \sin 2A \sin 3A}{\cos 3A}$ $= \cos 2A - \sin 2A \tan 3A$	1	4
		-----	1/2	
	c)	Prove that (in $\Delta ABC$ ), $\tan A + \tan B + \tan C = \tan A \tan B \tan C$		
	Ans.	<p>We have, <math>A + B + C = 180^\circ</math> or <math>\pi</math></p> <p><math>\therefore A + B = 180^\circ - C</math></p> <p><math>\therefore \tan(A + B) = \tan(180^\circ - C)</math></p> <p><math>\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C</math></p> <p><math>\therefore \tan A + \tan B = -\tan C [1 - \tan A \tan B]</math></p> <p><math>\therefore \tan A + \tan B = -\tan C + \tan A \tan B \tan C</math></p> <p><math>\therefore \tan A + \tan B + \tan C = \tan A \tan B \tan C</math></p>	1 1 1 1	
	d)	Prove that $\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$		4
	Ans.	<p><math>\tan 3\theta = \tan(\theta + 2\theta)</math></p> <p><math>= \frac{\tan \theta + \tan 2\theta}{1 - \tan \theta \tan 2\theta}</math></p> <p><math>= \frac{\tan \theta + \frac{2 \tan \theta}{1 - \tan^2 \theta}}{1 - \tan \theta \left( \frac{2 \tan \theta}{1 - \tan^2 \theta} \right)}</math></p> <p><math>= \frac{\tan \theta (1 - \tan^2 \theta) + 2 \tan \theta}{1 - \tan^2 \theta}</math></p> <p><math>= \frac{\tan \theta (1 - \tan^2 \theta - 2 \tan^2 \theta)}{1 - \tan^2 \theta}</math></p> <p><math>= \frac{\tan \theta - \tan^3 \theta + 2 \tan \theta}{1 - \tan^2 \theta - 2 \tan^2 \theta}</math></p> <p><math>= \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}</math></p>	1 1 1 1	
		-----		



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	e)	Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$		
	Ans.	$A = \cos^{-1}\left(\frac{4}{5}\right)$ $B = \cos^{-1}\left(\frac{12}{13}\right)$ $\therefore \cos A = \frac{4}{5}$ $\cos B = \frac{12}{13}$  $\cos(A+B) = \cos A \cos B - \sin A \sin B$ $= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}$ $= \frac{33}{65}$ $\therefore A+B = \cos^{-1}\left(\frac{33}{65}\right)$ $\therefore \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$ <hr/> $\cos(A+B) = \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}$ $= \frac{48}{65} - \frac{15}{65}$ $= \frac{33}{65}$ $\therefore A+B = \cos^{-1}\left(\frac{33}{65}\right)$ $\therefore \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$	1 1 1 1/2 1/2	4
	f)	If $\tan x = \frac{5}{6}$ , $\tan y = \frac{1}{11}$ , show that $x+y = \frac{\pi}{4}$		
	Ans.	$\tan x = \frac{5}{6}$ , $\tan y = \frac{1}{11}$ $\therefore x = \tan^{-1}\left(\frac{5}{6}\right)$ , $y = \tan^{-1}\left(\frac{1}{11}\right)$ $\therefore x+y = \tan^{-1}\left(\frac{5}{6}\right) + \tan^{-1}\left(\frac{1}{11}\right)$ $= \tan^{-1}\left(\frac{\frac{5}{6} + \frac{1}{11}}{1 - \left(\frac{5}{6}\right)\left(\frac{1}{11}\right)}\right)$ $= \tan^{-1}(1)$ $= \frac{\pi}{4}$	1 1 1 1	4









Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	b)	Prove that $\frac{\sin 4x + \sin 5x + \sin 6x}{\cos 4x + \cos 5x + \cos 6x} = \tan 5x$		
	Ans.	$\frac{\sin 4x + \sin 5x + \sin 6x}{\cos 4x + \cos 5x + \cos 6x} = \frac{\sin 4x + \sin 6x + \sin 5x}{\cos 4x + \cos 6x + \cos 5x}$ $= \frac{2 \sin 5x \cos(-x) + \sin 5x}{2 \cos 5x \cos(-x) + \cos 5x}$ $= \frac{\sin 5x [2 \cos(-x) + 1]}{\cos 5x [2 \cos(-x) + 1]}$ $= \tan 5x$	1+1 1 1	4
	c)	Prove that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right)$ , $x > 0$ , $y > 0$ , $xy < 1$		
Ans.	Put $\tan^{-1} x = A$ and $\tan^{-1} y = B$ $\therefore x = \tan A$ and $y = \tan B$ $\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $= \frac{x+y}{1-xy}$ $\therefore A+B = \tan^{-1} \left( \frac{x+y}{1-xy} \right)$ $\therefore \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right)$	1 1 1 1	4	
d)	Find the equation of a straight line passing through (2, 5) and the point of intersection of the lines $x+y=0$ , $2x-y=9$ .			
Ans.	$x+y=0$ $2x-y=9$ $\therefore 3x=9$ $\therefore x=3$ $y=-3$ $\therefore$ Point of intersection = (3, -3)		1 1	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		<p><math>\therefore</math> equation is,</p> $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$ $\therefore \frac{y - 5}{-3 - 5} = \frac{x - 2}{3 - 2}$ $\therefore 8x + y - 21 = 0$ <p style="text-align: center;"><b>OR</b></p> <p><math>\therefore</math> Point of intersection = (3, -3)</p> $\therefore \text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 5}{3 - 2} = -8$ <p><math>\therefore</math> equation is,</p> $y - y_1 = m(x - x_1)$ $\therefore y - 5 = -8(x - 2)$ $\therefore 8x + y - 21 = 0$ <p>-----</p>	1 1 OR 1  1/2 1/2	4
	e)	<p>Find the equation of the straight line passing through (-3, 10) and sum of their intercepts is 8.</p>		
	Ans.	<p>Let x-int = a                  y-int = b</p> $\therefore a + b = 8$ <p><math>\therefore</math> equation is</p> $\frac{x}{a} + \frac{y}{b} = 1 \quad \text{or} \quad \frac{x}{a} + \frac{y}{8 - a} = 1$ $\therefore bx + ay = ab$ $\therefore (8 - a)x + ay = a(8 - a)$ <p>But passing through (-3, 10)</p> $\therefore -3(8 - a) + 10a = a(8 - a)$ $\therefore -24 + 3a + 10a = 8a - a^2$ $\therefore a^2 + 5a - 24 = 0$ $\therefore a = 3, -8$ $\therefore \frac{x}{3} + \frac{y}{5} = 1 \quad \text{or} \quad \frac{x}{-8} + \frac{y}{16} = 1$ <p>-----</p>	1 1  1/2+1/2 1/2+1/2	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	f)	Find the acute angle between the lines $2x+3y=13$ , $2x-5y+7=0$		
	Ans.	For $2x+3y=13$ , $\text{slope } m_1 = -\frac{a}{b} = -\frac{2}{3}$ For $2x-5y+7=0$ , $\text{slope } m_2 = -\frac{a}{b} = -\frac{2}{-5} = \frac{2}{5}$ $\therefore \tan \theta = \frac{ m_1 - m_2 }{ 1 + m_1 \cdot m_2 }$ $= \frac{\left  -\frac{2}{3} - \frac{2}{5} \right }{\left  1 + \left( -\frac{2}{3} \right) \cdot \left( \frac{2}{5} \right) \right }$ $= \frac{16}{11} \quad \text{or} \quad 1.455$ $\therefore \theta = \tan^{-1} \left( \frac{16}{11} \right) \quad \text{or} \quad \tan^{-1}(1.455)$	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$	4
6)	a)	Attempt any FOUR of the following.  Find the equation of straight line passing through (5, 6) and making an angle $150^\circ$ with x-axis.		
	Ans.	Given $\theta = 150^\circ$ $\therefore \text{slope } m = \tan \theta = \tan 150^\circ$ $= -\frac{1}{\sqrt{3}}$ $\therefore \text{equation is}$ $y - y_1 = m(x - x_1)$ $\therefore y - 6 = -\frac{1}{\sqrt{3}}(x - 5)$ $\therefore \sqrt{3}y - 6\sqrt{3} = -x + 5$ $\therefore x + \sqrt{3}y - 6\sqrt{3} - 5 = 0$	1 1 1 1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks																						
6)		<b>OR</b>																								
		<p><math>\therefore</math> equation is</p> $y - y_1 = \tan \theta (x - x_1)$ <p><math>\therefore y - 6 = \tan 150^\circ (x - 5)</math></p> $\therefore y - 6 = -\frac{1}{\sqrt{3}}(x - 5)$ $\therefore \sqrt{3}y - 6\sqrt{3} = -x + 5$ $\therefore x + \sqrt{3}y - 6\sqrt{3} - 5 = 0$ <p>-----</p>	1 2 1	4																						
	b)	<p>If the length of perpendicular from (5, 4) on the straight line <math>2x + y + k = 0</math> is <math>4\sqrt{5}</math> units. Find the value of k.</p>																								
	Ans.	$p = \frac{ ax_1 + by_1 + c }{\sqrt{a^2 + b^2}}$ $\therefore 4\sqrt{5} = \frac{ 2(5) + 4 + k }{\sqrt{2^2 + 1^2}}$ $\therefore 4\sqrt{5} = \frac{ 14 + k }{\sqrt{5}}$ $\therefore 4\sqrt{5} \cdot \sqrt{5} =  14 + k $ $\therefore 20 =  14 + k $ $\therefore 20 = 14 + k \quad \text{or} \quad -20 = 14 + k$ $\therefore \boxed{6 = k} \quad \text{or} \quad \boxed{-34 = k}$ <p>-----</p>	1  1 $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$	4																						
	c)	<p>The scores of two batsmen A and B in ten innings during a certain season are as under:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>A</td><td>32</td><td>28</td><td>47</td><td>63</td><td>71</td><td>39</td><td>10</td><td>60</td><td>96</td><td>14</td> </tr> <tr> <td>B</td><td>19</td><td>31</td><td>48</td><td>53</td><td>67</td><td>90</td><td>10</td><td>62</td><td>40</td><td>80</td> </tr> </table> <p>Find which of the two batsmen is more consisting in scoring (use coefficient of variance).</p>	A	32	28	47	63	71	39	10	60	96	14	B	19	31	48	53	67	90	10	62	40	80		
A	32	28	47	63	71	39	10	60	96	14																
B	19	31	48	53	67	90	10	62	40	80																



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No. of Students	10	15	16	20	21	22	09	08														
	Ans.	$L = 99 \quad S = 20$ Difference between two sets = $D = 1$ $\therefore \text{Range} = L - S + D$ $= 99 - 20 + 1$ $= 80$	1 1																			
		$\text{Coeff. of Range} = \frac{L - S + D}{L + S}$ $= \frac{99 - 20 + 1}{99 + 20}$ $= \frac{80}{119} \quad \text{or} \quad 0.672$	1 1	4																		
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6)		$A = 42.5, \quad h = 5, \quad d_i = \frac{x_i - A}{h}$ $\therefore \bar{x} = A + \frac{\sum f_i d_i}{N} \times h$ $= 42.5 + \frac{-115}{405} \times 5$ $= 41.08$ $S.D. = h \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$ $= 5 \times \sqrt{\frac{1625}{405} - \left(\frac{-115}{405}\right)^2}$ $= 9.914$ $\therefore \text{Variance} = (S.D.)^2$ $= 9.914^2$ $= 98.287$ $\text{Coeff. of Variance} = \frac{S.D.}{\bar{x}} \times 100$ $= \frac{9.914}{41.08} \times 100$ $= 24.133$ <p style="text-align: center;"><b>OR</b></p> $\therefore \text{Variance} = h^2 \left[ \frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2 \right]$ $= 5^2 \left[ \frac{1625}{405} - \left(\frac{-115}{405}\right)^2 \right]$ $= 98.287$ $\text{Coeff. of Variance} = \frac{\sqrt{\text{variance}}}{\bar{x}} \times 100$ $= \frac{\sqrt{98.287}}{41.08} \times 100$ $= 24.133$	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>OR</p> <p>1</p> <p>1</p>	4
		<p><b>Important Note</b></p> <p>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. <b>In such case, FIRST SEE whether the method falls within the scope of the curriculum, and THEN ONLY give appropriate marks in accordance with the scheme of marking.</b></p>		