SUMMER – 2016 EXAMINATION

MODEL ANSWER

Subject: APPLIED MATHEMATICS (AMS)

Subject Code: 17301

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.

2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.

3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)

4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.

5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and the model answer.

6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.

7) For programming language papers, credit may be given to any other program based on equivalent concept.

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		**Attempt any <u>TEN</u> of the following:**		20
	a)	Find the radius of curvature of the curve $y = x^3$ at $(2,8)$		
	Ans	$y = x^3$		
		$\frac{dy}{dx} = 3x^2$		
		$\frac{d^2y}{dx^2} = 6x$	½	
		$\therefore$ at $(2,8)$		
		$\frac{dy}{dx} = 12$		
		$\frac{d^2y}{dx^2} = 12$	½	
		$\therefore$ Radius of curvature is $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$		
		$\therefore \rho = \frac{\left[1+(12)^2\right]^{\frac{3}{2}}}{12}$	½	
		$\therefore \rho = 145.50$	½	02
		---		
	b)	At which point on the curve $y = 3x - x^2$, the slope of tangent is -5?		
	Ans	$y = 3x - x^2$		
		$\frac{dy}{dx} = 3 - 2x$	½	
		$\therefore m = 3 - 2x$		
		$-5 = 3 - 2x$		
		$2x = 8$	½	
		$x = 4$		
		$\therefore y = -4$	½	02
		$\therefore$ point is $(4,-4)$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	c)	Evaluate: $\int \frac{2x^3+5x^2+4}{\sqrt{x}}\,dx$		
	Ans	$\int \frac{2x^3+5x^2+4}{\sqrt{x}}\,dx$		
		$= \int \left(2x^3+5x^2+4\right)x^{-\frac{1}{2}}\,dx$	½	
		$= \int \left(2x^{\frac{5}{2}}+5x^{\frac{3}{2}}+4x^{-\frac{1}{2}}\right)dx$	½	
		$= \frac{4}{7}x^{\frac{7}{2}}+2x^{\frac{5}{2}}+8x^{\frac{1}{2}}+c$	1	02
		--		
	d)	Evaluate: $\int \sin^2 2x\,dx$		
	Ans	$\int \sin^2 2x\,dx$		
		$= \frac{1}{2}\int \left(1-\cos 4x\right)\,dx$	1	
		$= \frac{1}{2}\left(x-\frac{\sin 4x}{4}\right)+c$	1	02
		--		
	e)	Evaluate: $\int \frac{1}{x\log x}\,dx$		
	Ans	$\int \frac{1}{x\log x}\,dx$		
		Put $\log x = t$		
		$\therefore \frac{1}{x}dx = dt$	½	
		$= \int \frac{1}{t}\,dt$	½	
		$= \log t + c$	½	
		$= \log\left(\log x\right)+c$	½	02
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	f)	Evaluate: $\int x^2 e^x \, dx$		
	Ans	$\int x^2 e^x \, dx$		
		$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2 \right) dx$	½	
		$= x^2 e^x - \int e^x 2x \, dx$	½	
		$= x^2 e^x - 2 \int e^x x \, dx$		
		$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$	½	
		$= x^2 e^x - 2 \left[x e^x - e^x \right] + c$	½	02
		--		
	g)	Evaluate: $\int_2^4 \frac{1}{2x+3} dx$		
	Ans	$\int_2^4 \frac{1}{2x+3} dx$		
		$= \left[\frac{\log(2x+3)}{2} \right]_2^4$	1	
		$= \frac{1}{2} [\log 11 - \log 7]$	½	
		$= \frac{1}{2} \log \left(\frac{11}{7} \right)$	½	02
		--		
	h)	Find the area under the curve $y = e^x$ from the ordinate $x = 0$ to $x = 1$		
	Ans	$A = \int_a^b y dx$		
		$= \int_0^1 e^x dx$	½	
		$= \left[e^x \right]_0^1$	1	
		$= e - 1$	½	02
		--		
	i)	Find the order and degree of the equation $\left[1 + \left(\frac{dy}{dx} \right)^3 \right]^{\frac{5}{3}} = 2 \frac{d^2 y}{dx^2}$		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2.		$\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{5}{3}} = 2\frac{d^2y}{dx^2}$		
		$\therefore \left[1+\left(\frac{dy}{dx}\right)^3\right]^5 = 8\left(\frac{d^2y}{dx^2}\right)^3$		
		$\therefore Order = 2$	1	
		$Degree = 3$	1	02
		--		
	j)	Find the differential equation from the relation $y = Ae^{mx}$		
	Ans	$y = Ae^{mx}$		
		$\therefore \frac{dy}{dx} = mAe^{mx}$	1	
		$\therefore \frac{dy}{dx} = my$	1	
		$or \ \frac{dy}{dx} - my = 0$		02
		--		
	k)	Three fair coins are tossed. Find the probability that atleast two heads appear.		
	Ans	$S = \{HHH, HTT, THT, TTH, HTH, HHT, THH, TTT\}$		
		$\therefore n(S) = 8$	½	
		atleast two heads		
		$A = \{HHH, HTH, HHT, THH\}$		
		$n(A) = 4$	½	
		$\therefore p(A) = \frac{n(A)}{n(S)} = \frac{4}{8}$	½	
		$\therefore p(A) = \frac{1}{2}$ or 0.5	½	02
		--		
	l)	Five men in a company of 20 are graduate. If 3 men are picked up out of 20 at random, what is probability that they all are graduates?		
	Ans		½	
		$n(S) = {}^{20}C_3 = 1140$		
		$n(A) = {}^{5}C_3 = 10$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.		$\therefore\ p(A)=\frac{n(A)}{n(S)}$		
		$=\frac{10}{1140}$ or 0.00877	1	02
		--		
2.		**Attempt any FOUR of the following:**		**16**
	a)	Evaluate: $\int\frac{(\tan^{-1}x)^3}{1+x^2}dx$		
	Ans	$\int\frac{(\tan^{-1}x)^3}{1+x^2}dx$		
		put $\tan^{-1}x=t$		
		$\therefore\ \frac{1}{1+x^2}dx=dt$	1	
		$=\int t^3dt$	1	
		$=\frac{t^4}{4}+c$	1	
		$=\frac{(\tan^{-1}x)^4}{4}+c$	1	04
		--		
	b)	Evaluate: $\int\frac{x}{\sqrt{9+8x-x^2}}\,dx$		
	Ans	$\int\frac{x}{\sqrt{9+8x-x^2}}\,dx$		
		$=-\frac{1}{2}\int\frac{8-2x-8}{\sqrt{9+8x-x^2}}\,dx$	1	
		$=-\frac{1}{2}\left[\int\frac{8-2x}{\sqrt{9+8x-x^2}}\,dx-8\int\frac{1}{\sqrt{9+8x-x^2}}\,dx\right]$	½	
		$=-\frac{1}{2}\left[2\sqrt{9+8x-x^2}-8\int\frac{1}{\sqrt{9+16-16+8x-x^2}}\,dx\right]$	1	
		$=-\left[\sqrt{9+8x-x^2}-4\int\frac{1}{\sqrt{25-(16-8x+x^2)}}\,dx\right]$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$= -\left[\sqrt{9+8x-x^2} - 4\int \frac{1}{\sqrt{(5)^2-(x-4)^2}}\,dx\right]$	½	
		$= -\left[\sqrt{9+8x-x^2} - 4\sin^{-1}\left(\frac{x-4}{5}\right)\right] + c$	1	04
		--		
	c)	Evaluate $\int x\tan^{-1}x\,dx$		
	Ans	$\int x\tan^{-1}x\,dx$		
		$= \tan^{-1}x\int x\,dx - \int\left(\int x\,dx\,\frac{d}{dx}\left(\tan^{-1}x\right)\right)dx$	1	
		$= \tan^{-1}x\,\frac{x^2}{2} - \int \frac{1}{1+x^2}\,\frac{x^2}{2}\,dx$	1	
		$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}\,dx$	½	
		$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx$	½	
		$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left(x-\tan^{-1}x\right) + c$	1	04
		--		
	d)	Find the maximum and minimum value of $y = x^3 - 9x^2 + 24x$		
	Ans	Let $y = x^3 - 9x^2 + 24x$		
		$\therefore \frac{dy}{dx} = 3x^2 - 18x + 24$	½	
		$\therefore \frac{d^2y}{dx^2} = 6x - 18$	½	
		Consider $\frac{dy}{dx} = 0$		
		$3x^2 - 18x + 24 = 0$	½	
		$\therefore x = 2$ or $x = 4$	½	
		at $x = 2$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$\frac{d^2y}{dx^2}=6(2)-18=-6<0$ $\therefore$ y is maximum at $x=2$ $y_{max}=2^3-9(2)^2+24(2)$ $=20$ at $x=4$ $\frac{d^2y}{dx^2}=6(4)-18=6>0$ $\therefore$ y is minimum at $x=4$ $y_{min}=4^3-9(4)^2+24(4)$ $=16$	½ ½ ½ ½	04
	e)	Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=1$ at $\left(\frac{1}{4},\frac{1}{4}\right)$		
	Ans	$\sqrt{x}+\sqrt{y}=1$ $\therefore\ \frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$ $\therefore\ \frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}$ $\frac{d^2y}{dx^2}=\frac{-\left[\sqrt{x}\frac{1}{2\sqrt{y}}\frac{dy}{dx}-\sqrt{y}\frac{1}{2\sqrt{x}}\right]}{\left(\sqrt{x}\right)^2}$ $\frac{d^2y}{dx^2}=\frac{-\left[\sqrt{x}\frac{1}{2\sqrt{y}}\left(-\frac{\sqrt{y}}{\sqrt{x}}\right)-\sqrt{y}\frac{1}{2\sqrt{x}}\right]}{x}$ $\frac{d^2y}{dx^2}=\frac{-\left[-\frac{1}{2}-\frac{\sqrt{y}}{2\sqrt{x}}\right]}{x}$ $\therefore$ at $\left(\frac{1}{4},\frac{1}{4}\right)$ $\frac{dy}{dx}=-\frac{\sqrt{\frac{1}{4}}}{\sqrt{\frac{1}{4}}}=-1$	½ ½ ½ ½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2.		$\frac{d^2y}{dx^2}=\frac{-\left[-\frac{1}{2}-\frac{1}{2}\right]}{\frac{1}{4}}=4$	½	
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$		
		$\therefore\ \rho=\frac{\left[1+(-1)^2\right]^{\frac{3}{2}}}{4}$	1	
		$\therefore\ \rho=0.707$	½	04
		--		
	f)	Show that equation of tangent to $\left(\frac{x}{a}\right)^m+\left(\frac{y}{b}\right)^m=2$ at the point (a,b) is $\frac{x}{a}+\frac{y}{b}=2$		
	Ans	$\left(\frac{x}{a}\right)^m+\left(\frac{y}{b}\right)^m=2$		
		$\therefore\ m\left(\frac{x}{a}\right)^{m-1}\frac{1}{a}+m\left(\frac{y}{b}\right)^{m-1}\frac{1}{b}\frac{dy}{dx}=0$	½	
		$\therefore\ b\left(\frac{x}{a}\right)^{m-1}+a\left(\frac{y}{b}\right)^{m-1}\frac{dy}{dx}=0$	½	
		at (a,b)		
		$\therefore\ b\left(\frac{a}{a}\right)^{m-1}+a\left(\frac{b}{b}\right)^{m-1}\frac{dy}{dx}=0$		
		$b+a\frac{dy}{dx}=0$		
		$\therefore\ \frac{dy}{dx}=-\frac{b}{a}$		
		$\therefore$ slope is $m_1=-\frac{b}{a}$	1	
		equation of tangent at (a,b) is		
		$y-y_1=m_1(x-x_1)$		
		$y-b=-\frac{b}{a}(x-a)$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		$ay - ab = -bx + ab$		
		$bx + ay = 2ab$		
		$\therefore \frac{x}{a} + \frac{y}{b} = 2$	1	04
		--		
3.		**Attempt any <u>FOUR</u> of the following:**		**16**
	a)	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{5 + 4\cos x}$		
	Ans	$\int_0^{\frac{\pi}{2}} \frac{dx}{5 + 4\cos x}$		
		Put $\tan\frac{x}{2} = t$		
		$\cos x = \frac{1 - t^2}{1 + t^2}$, $dx = \frac{2dt}{1 + t^2}$	½	
		when $x \to 0$ to $\frac{\pi}{2}$ $t \to 0$ to 1	½	
		$\therefore \int_0^1 \frac{1}{5 + 4\left(\frac{1 - t^2}{1 + t^2}\right)} \frac{2dt}{1 + t^2}$	½	
		$= 2\int_0^1 \frac{1}{5\left(1 + t^2\right) + 4\left(1 - t^2\right)} dt$		
		$= 2\int_0^1 \frac{1}{5 + 5t^2 + 4 - 4t^2} dt$		
		$= 2\int_0^1 \frac{1}{9 + t^2} dt$	½	
		$= 2\int_0^1 \frac{1}{(3)^2 + t^2} dt$		
		$= \frac{2}{3}\left[\tan^{-1}\left(\frac{t}{3}\right)\right]_0^1$	1	
		$= \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\right)$	1	04
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.	b)	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x \cos x}\, dx$		
	Ans	$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x \cos x}\, dx$		
		$I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x \cos x}\, dx ----------- (1)$		
		$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right) - \cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}\, dx$	1	
		$I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1+\cos x \sin x}\, dx ----------- (2)$	1	
		add (1) and (2)		
		$I + I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x \cos x}\, dx + \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1+\cos x \sin x}\, dx$	½	
		$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sin x - \cos x}{1+\sin x \cos x} + \frac{\cos x - \sin x}{1+\cos x \sin x}\right) dx$	½	
		$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sin x - \cos x + \cos x - \sin x}{1+\sin x \cos x}\right) dx$	½	
		$2I = 0$	½	
		$\therefore I = 0$		04
		--		
	c)	Find by integration the area of the ellipse $4x^2 + 9y^2 = 36$		
	Ans	$4x^2 + 9y^2 = 36$		
		$\therefore \frac{x^2}{9} + \frac{y^2}{4} = 1$		
		$\therefore y^2 = \frac{4}{9}\left(9 - x^2\right)$		
		$\therefore y = \frac{2}{3}\sqrt{9 - x^2}$	1	
		area, $A = 4\int_a^b y\,dx$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.		$A = 4\left[\frac{2}{3}\int_0^3 \sqrt{(3)^2 - x^2}\,dx\right]$	1	
		$A = \frac{8}{3}\left[\frac{x}{2}\sqrt{(3)^2 - x^2} + \frac{(3)^2}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$	1	
		$A = \frac{8}{3}\left[\frac{9}{2}\sin^{-1}(1) - 0\right]$	½	
		$A = 12\frac{\pi}{2}$		
		$A = 6\pi$	½	04
		--		
	d)	Find the perticular solution of D.E. $y\sqrt{1-x^2}\,dy + x\sqrt{1-y^2}\,dx = 0$		
		when $x = \frac{3}{4}, y = \frac{4}{5}$		
	Ans	$y\sqrt{1-x^2}\,dy + x\sqrt{1-y^2}\,dx = 0$		
		$y\sqrt{1-x^2}\,dy = -x\sqrt{1-y^2}\,dx$		
		$\therefore \frac{y}{\sqrt{1-y^2}}\,dy = -\frac{x}{\sqrt{1-x^2}}\,dx$		
		$\therefore \int \frac{y}{\sqrt{1-y^2}}\,dy = -\int \frac{x}{\sqrt{1-x^2}}\,dx$	1	
		$\therefore \frac{-1}{2}2\sqrt{1-y^2} = \frac{1}{2}2\sqrt{1-x^2} + c$	1	
		$\therefore -\sqrt{1-y^2} = \sqrt{1-x^2} + c$	½	
		when $x = \frac{3}{4}, y = \frac{4}{5}$		
		$\therefore -\sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\left(\frac{3}{4}\right)^2} + c$	½	
		$\therefore \frac{-3}{5} = \frac{\sqrt{7}}{4} + c$		
		$\therefore c = -\left(\frac{3}{5} + \frac{\sqrt{7}}{4}\right)$	½	
		$-\sqrt{1-y^2} = \sqrt{1-x^2} - \left(\frac{3}{5} + \frac{\sqrt{7}}{4}\right)$	½	04
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.	e)	Solve the D.E. $\frac{dy}{dx}=\frac{(x+y)^2}{xy}$		
	Ans	$\frac{dy}{dx}=\frac{(x+y)^2}{xy}$		
		Put $y=vx$		
		$\frac{dy}{dx}=v+x\frac{dv}{dx}$	½	
		$\therefore v+x\frac{dv}{dx}=\frac{(x+vx)^2}{x(vx)}$	½	
		$v+x\frac{dv}{dx}=\frac{x^2(1+v)^2}{vx^2}$		
		$v+x\frac{dv}{dx}=\frac{(1+v)^2}{v}$		
		$x\frac{dv}{dx}=\frac{1+2v+v^2}{v}-v$		
		$x\frac{dv}{dx}=\frac{1+2v+v^2-v^2}{v}$		
		$x\frac{dv}{dx}=\frac{1+2v}{v}$		
		$\frac{v}{1+2v}dv=\frac{1}{x}dx$	½	
		$\int\frac{v}{1+2v}dv=\int\frac{1}{x}dx$	½	
		$\frac{1}{2}\int\frac{1+2v-1}{1+2v}dv=\log x+c$	½	
		$\frac{1}{2}\int\left(\frac{1+2v}{1+2v}-\frac{1}{1+2v}\right)dv=\log x+c$	½	
		$\frac{1}{2}\int\left(1-\frac{1}{1+2v}\right)dv=\log x+c$	½	
		$\frac{1}{2}\left(v-\frac{1}{2}\log(1+2v)\right)=\log x+c$		
		$\frac{1}{2}\left(\frac{y}{x}-\frac{1}{2}\log\left(1+\frac{2y}{x}\right)\right)=\log x+c$	½	04
		$\frac{y}{x}-\log\left(\frac{x+2y}{x}\right)=2\log x+2c$		
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3.	f)	Solve the D.E. $x\frac{dy}{dx}-y=x^2\cos^2 x$		
	Ans	$\frac{dy}{dx}-\frac{y}{x}=x\cos^2 x$		
		$\therefore\ P=-\frac{1}{x}$ and $Q=x\cos^2 x$	½	
		$IF=e^{-\int\frac{1}{x}dx}=e^{-\log x}=\frac{1}{x}$	½	
		$\therefore\ yIF=\int QIF\,dx+c$		
		$y\frac{1}{x}=\int x\cos^2 x\frac{1}{x}dx+c$	1	
		$\frac{y}{x}=\int\cos^2 x\,dx+c$		
		$\frac{y}{x}=\frac{1}{2}\int(1-\cos 2x)dx+c$	1	
		$\frac{y}{x}=\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+c$	1	04
		--		
4.		**Attempt any <u>FOUR</u> of the following:**		**16**
	a)	Evaluate: $\int\frac{x}{(x^2-1)(x^2+2)}dx$		
	Ans	$\int\frac{x}{(x^2-1)(x^2+2)}dx$		
		Put $x^2=t$		
		$2xdx=dt$		
		$xdx=\frac{dt}{2}$		
		$=\frac{1}{2}\int\frac{dt}{(t-1)(t+2)}$	1	
		Let $\frac{1}{(t-1)(t+2)}=\frac{A}{t-1}+\frac{B}{t+2}$		
		$1=A(t+2)+B(t-1)$		
		Put $t=-2$		
		$1=B(-3)$		
		$B=-\frac{1}{3}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		Put $t = 1$ $1 = A(3)$ $A = \frac{1}{3}$	½	
		$\therefore \frac{1}{(t-1)(t+2)} = \frac{\frac{1}{3}}{t-1} + \frac{-\frac{1}{3}}{t+2}$		
		$\therefore \frac{1}{2}\int \frac{dt}{(t-1)(t+2)} = \frac{1}{2}\int \left(\frac{\frac{1}{3}}{t-1} + \frac{-\frac{1}{3}}{t+2} \right) dt$	½	
		$= \frac{1}{6}\log(t-1) - \frac{1}{6}\log(t+2) + c$	1	
		$= \frac{1}{6}\log(x^2-1) - \frac{1}{6}\log(x^2+2) + c$	½	
		or $= \frac{1}{6}\log\left(\frac{x^2-1}{x^2+2}\right) + c$		04
		--		
	b) Ans	Evaluate: $\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}}\, dx$		
		$I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}}\, dx$ --------------------(1)		
		$I = \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{7-(7-x)}}\, dx$	1	
		$I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{7-x} + \sqrt[3]{x}}\, dx$ --------------------(2)		
		add (1) and (2)		
		$I + I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}}\, dx + \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}}\, dx$	1	
		$2I = \int_0^7 \frac{\sqrt[3]{x} + \sqrt[3]{7-x}}{\sqrt[3]{x} + \sqrt[3]{7-x}}\, dx$		
		$2I = \int_0^7 1\, dx$		
		$2I = [x]_0^7$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		$2I = 7 - 0$	½	
		$I = \frac{7}{2} = 3.5$	½	04
		--		
	c)	Find the area enclosed between the parabola $y = x^2$ and the line $y = 4$		
	Ans	$A = \int_a^b y\ dx$		
		$A = \int_{-2}^{2} x^2 dx$	1	
		$= \left[\frac{x^3}{3}\right]_{-2}^{2}$	1	
		$= \left[\frac{(2)^3}{3}\right] - \left[\frac{(-2)^3}{3}\right]$	1	
		$= \frac{16}{3}$ or 5.33	1	04
		--		
	d)	A problem is given to the three students Sumit, Amit and Akbar, whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If they attempt to solve a problem independently, find the probability that the problem is solved by atleast one of them.		
	Ans	Given Probability of Sumit $P(A) = \frac{1}{2}$ $\therefore P(A') = 1 - \frac{1}{2} = \frac{1}{2}$		
		Probability of Sumit $P(B) = \frac{1}{3}$ $\therefore P(B') = 1 - \frac{1}{3} = \frac{2}{3}$		
		Probability of Sumit $P(C) = \frac{1}{4}$ $\therefore P(C') = 1 - \frac{1}{4} = \frac{3}{4}$	1½	
		$= P(A \cup B \cup C)$		
		$= 1 - P(A \cup B \cup C)'$	½	
		$= 1 - P(A' \cap B' \cap C')$	½	
		$= 1 - \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}\right)$	½	

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
4.		$= 1 - \frac{1}{4}$	½	
		$= \frac{3}{4} \quad or \quad 0.75$	½	04
		--		
	e)	In sampling a large number of parts manufactured by a machine , the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain atleast 3 defective parts.		
	Ans	$Given \quad n = 1000$		
		$m = 2$		
		r = at least three		
		$= 3, 4, 5 \ldots$		
		$\therefore \; p(r) = \frac{e^{-m} m^{r}}{r!}$		
		$p(r) \; = 1 - [\, p(0) + p(1) + p(2)]$	1	
		$= 1 - \left[\frac{e^{-2}(2)^{0}}{0!} + \frac{e^{-2}(2)^{1}}{1!} + \frac{e^{-2}(2)^{2}}{2!} \right]$	2	
		$= 1 - [0.1353 + 0.2706 + 0.2706]$		
		$= 1 - 0.6765$		
		$= 0.3235$	1	04
		--		
	f)	In a certain examination 500 students appered . Mean score is 68 with S.D. 8. Find the number of students scoring		
		i) Less than 50		
		ii) More than 60		
		(Given that area between $z = 0$ to $z = 2.25$ is 0.4878 and area between $z = 0$ to $z = 1$ is 0.3413)		
	Ans	Given $\bar{x} = 68, \; \sigma = 8$		
		Standard normal variate, $Z = \frac{x - \bar{x}}{\sigma}$		
		i) For $x = 50$, $Z = \frac{50 - 68}{8} = -2.25$	½	
		$p = (\text{area less than } -2.25) = 0.5 - A(-2.25)$	½	
		$= 0.5 - 0.4878$		
		$= 0.0122$	½	
		$\therefore$ number of students $= 500 \times 0.0122 = 6.1 \approx 6$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4.		ii) For $x = 60$, $Z = \frac{60-68}{8} = -1$	½	
		$p = (\text{area more than } -1) = 0.5 + A(-1)$	½	
		$= 0.5 + 0.3413$	½	
		$= 0.8413$		
		$\therefore$ number of students $= 500 \times 0.8413 = 420.65 \approx 421$	½	04

5.		**Attempt any <u>FOUR</u> of the following:**		**16**
	a)	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$		
	Ans	$\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$		
		Put $\cos x = t$		
		$-\sin x dx = dt$	½	
		$\sin x dx = -dt$ when $x \to 0$ to $\frac{\pi}{2}$, $t \to 1$ to 0	½	
		$\therefore I = -\int_1^0 \frac{t}{t^2 + 3t + 2} dt$	½	
		$\therefore I = -\int_1^0 \frac{t}{(t+1)(t+2)} dt$		
		Let $\frac{t}{(t+1)(t+2)} = \frac{A}{t+1} + \frac{B}{t+2}$		
		$t = A(t+2) + B(t+1)$		
		Put $t = -1$		
		$-1 = A(1)$		
		$\therefore A = -1$	½	
		Put $t = -2$		
		$-2 = B(-1)$		
		$\therefore B = 2$	½	
		$\frac{t}{(t+1)(t+2)} = \frac{-1}{t-1} + \frac{2}{t+1}$		
		$\therefore -\int_1^0 \frac{t}{(t+1)(t+2)} dt = -\int_1^0 \left(\frac{-1}{t+1} + \frac{2}{t+2} \right) dt$		
		$= \left[\log(t+1) - 2\log(t+2) \right]_1^0$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.		$= \left[\left(\log 1 - 2\log 2 \right) - \left(\log 2 - 2\log 3 \right) \right]$		
		$= -3\log 2 + 2\log 3$	½	04
		$= 2\log 3 - 3\log 2$		
		--		
	b)	Evaluate: $\int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right) dx$		
	Ans	$\int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right) dx$		
		$= \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right) dx$	½	
		$= \int_0^{\frac{\pi}{4}} \log\left(1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right) dx$	½	
		$= \int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-\tan x}{1+\tan x}\right) dx$	½	
		$= \int_0^{\frac{\pi}{4}} \log\left(\frac{1+\tan x+1-\tan x}{1+\tan x}\right) dx$		
		$= \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan x}\right) dx$	½	
		$= \int_0^{\frac{\pi}{4}} \left[\log 2 - \log\left(1+\tan x\right)\right] dx$	½	
		$= \log 2 \int_0^{\frac{\pi}{4}} dx - \int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right) dx$		
		$\therefore I = \log 2 \left[x\right]_0^{\frac{\pi}{4}} - I$	½	
		$2I = \log 2\left[\frac{\pi}{4}-0\right]$	½	
		$I = \frac{\pi}{8}\log 2$	½	04
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.	c) Ans	Find the area enclosed between the parabolas $y^2 = 4x$ and $x^2 = 4y$		
		Given $y^2 = 4x$		
		$x^2 = 4y$		
		$\therefore y = \frac{x^2}{4}$		
		$\therefore \left(\frac{x^2}{4}\right)^2 = 4x$		
		$\therefore x^4 - 64x = 0$		
		$\therefore x = 0, 4$	1	
		$A = \int_a^b (y_1 - y_2)dx$		
		$\therefore A = \int_0^4 \left(2x^{\frac{1}{2}} - \frac{x^2}{4}\right) dx$	1	
		$\therefore A = \left(\frac{4}{3}x^{\frac{3}{2}} - \frac{x^3}{12}\right)_0^4$	1	
		$\therefore A = \frac{32}{3} - \frac{16}{3}$		
		$\therefore A = \frac{16}{3}$ or 5.33	1	04
		--		
	d) Ans	Solve $\frac{dy}{dx} = \cos(x+y)$		
		Put $x + y = v$	½	
		$1 + \frac{dy}{dx} = \frac{dv}{dx}$	½	
		$\frac{dy}{dx} = \frac{dv}{dx} - 1$		
		$\therefore \frac{dv}{dx} - 1 = \cos v$		
		$\therefore \frac{dv}{dx} = 1 + \cos v$		
		$\therefore \frac{1}{1+\cos v} dv = dx$		
		$\therefore \int \frac{1}{1+\cos v} dv = \int dx$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.		$\therefore \int \frac{1}{2\cos^2\left(\frac{v}{2}\right)} dv = x + c$	½	
		$\therefore \frac{1}{2}\int \sec^2\left(\frac{v}{2}\right) dv = x + c$	½	
		$\therefore \frac{1}{2} \frac{\tan\left(\frac{v}{2}\right)}{\frac{1}{2}} = x + c$	½	
		$\therefore \tan\left(\frac{v}{2}\right) = x + c$	½	
		$\therefore \tan\left(\frac{x+y}{2}\right) = x + c$	½	04
		OR		
		Put $x + y = v$	½	
		$1 + \frac{dy}{dx} = \frac{dv}{dx}$	½	
		$\frac{dy}{dx} = \frac{dv}{dx} - 1$		
		$\therefore \frac{dv}{dx} - 1 = \cos v$		
		$\therefore \frac{dv}{dx} = 1 + \cos v$		
		$\therefore \frac{1}{1+\cos v} dv = dx$		
		$\therefore \int \frac{1}{1+\cos v} dv = \int dx$	½	
		Put $\tan\frac{v}{2} = t$		
		$dv = \frac{2dt}{1+t^2}$		
		$\cos v = \frac{1-t^2}{1+t^2}$	½	
		$\therefore \int \frac{1}{1+\frac{1-t^2}{1+t^2}} \frac{2dt}{1+t^2} = x + c$	½	
		$\therefore 2\int \frac{1}{1+t^2+1-t^2} dt = x + c$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.		$\therefore 2\int \frac{1}{2}dt = x + c$		
		$\therefore t = x + c$	½	
		$\therefore \tan\left(\frac{v}{2}\right) = x + c$	½	
		$\therefore \tan\left(\frac{x+y}{2}\right) = x + c$	½	04
		OR		
		Put $x + y = v$		
		$1 + \frac{dy}{dx} = \frac{dv}{dx}$	½	
		$\frac{dy}{dx} = \frac{dv}{dx} - 1$	½	
		$\therefore \frac{dv}{dx} - 1 = \cos v$		
		$\therefore \frac{dv}{dx} = 1 + \cos v$		
		$\therefore \frac{1}{1+\cos v}dv = dx$		
		$\therefore \int \frac{1}{1+\cos v}dv = \int dx$	½	
		$\therefore \int \frac{1-\cos v}{1-\cos^2 v}dv = \int dx$	½	
		$\therefore \int \frac{1-\cos v}{\sin^2 v}dv = x + c$	½	
		$\therefore \int\left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right)dv = x + c$		
		$\therefore \int\left(\operatorname{cosec}^2 v - \cot v \operatorname{cosec} v\right)dv = x + c$	½	
		$\therefore -\cot v + \operatorname{cosec} v = x + c$	½	
		$\therefore -\cot(x+y) + \operatorname{cosec}(x+y) = x + c$	½	04
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5.	e)	Solve D.E. $\left(2xy+y^2\right)dx+\left(x^2+2xy+\sin y\right)dy=0$		
	Ans	$M=2xy+y^2\ ,\ N=x^2+2xy+\sin y$		
		$\frac{\partial M}{\partial y}=2x+2y\ ,\ \frac{\partial N}{\partial x}=2x+2y$	1	
		$\therefore\ \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$		
		$\therefore$ equation is an exact D.E.	1	
		$\therefore$ Solution is		
		$\int\limits_{y-constant} M\,dx+\int\limits_{\substack{terms\ free\\ from\ x}} N\,dy=c$		
		$\therefore\ \int\limits_{y-constant}\left(2x+2y\right)dx+\int \sin y\,dy=c$	1	04
		$x^2+2xy-\cos y=c$	1	
		--		
	f)	Solve D.E. $x\frac{dy}{dx}-y=x^2$		
	Ans	$x\frac{dy}{dx}-y=x^2$		
		$\therefore\ \frac{dy}{dx}-\frac{1}{x}y=x$	½	
		$\therefore\ P=-\frac{1}{x}$ and $Q=x$	½	
		$IF=e^{-\int\frac{1}{x}dx}=e^{-\log x}=\frac{1}{x}$	1	
		$\therefore\ yIF=\int Q\,IF\,dx+c$		
		$y\frac{1}{x}=\int x\frac{1}{x}dx+c$	1	
		$\frac{y}{x}=\int dx+c$		
		$\frac{y}{x}=x+c$	1	04
		$or\ y=x^2+cx$		
		--		
6.		**Attempt any <u>FOUR</u> of the following:**		**16**
	a)	Divide 80 into two parts such that their product is maximum.		
	Ans	consider x and y be the two parts.		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		$\therefore x+y=80$		
		$y=80-x$		
		product is, $P=xy$		
		$P=x(80-x)$		
		$P=80x-x^2$	1	
		$\frac{dP}{dx}=80-2x$	1	
		Let $\frac{dP}{dx}=0$		
		$\therefore 80-2x=0$		
		$x=40$	½	
		$\frac{d^2P}{dx^2}=-2$	1	
		$\therefore P$ is maximum at $x=40$		
		$\therefore x=40, y=40$	½	04
		--		
	b)	Find the equation of tangent and normal to the curve $4x^2+9y^2=40$ at point $(1,2)$		
	Ans	$4x^2+9y^2=40$		
		$\therefore 8x+18y\frac{dy}{dx}=0$		
		$\frac{dy}{dx}=\frac{-8x}{18y}=\frac{-4x}{9y}$	1	
		at point $(1,2)$		
		$\frac{dy}{dx}=$ slope of tangent $=\frac{-4}{18}=\frac{-2}{9}$	½	
		slope of normal $=\frac{9}{2}$	½	
		Equation of tangent at $(1,2)$ is		
		$y-2=\frac{-2}{9}(x-1)$		
		$\therefore 2x+9y-20=0$	1	
		Equation of normal at $(1,2)$ is		
		$y-2=\frac{9}{2}(x-1)$		
		$\therefore 9x-2y-5=0$	1	04

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.	c)	Solve the D.E. $\frac{dy}{dx} = e^{x-y} + xe^{-y}$		
	Ans	$\frac{dy}{dx} = \left(e^{x} + x\right)e^{-y}$	1	
		$e^{y}dy = \left(e^{x} + x\right)dx$	1	
		$\int e^{y}dy = \int \left(e^{x} + x\right)dx$	1	
		$e^{y} = e^{x} + \frac{x^{2}}{2} + c$	1	04
		--		
	d)	A box contain 7 red, 5 white and 8 green balls identical in all respect except colour. One ball is drawn at ramdom. Find the probility that it is not white.		
	Ans	Total number of balls $n = 20$		
		$n(S) = {}^{20}C_{1} = 20$	1	
		$n(A) = {}^{15}C_{1} = 15$	1	
		$P(A) = \frac{n(A)}{n(S)} = \frac{15}{20} = 0.75$	2	04
		--		
	e)	Two unbaised dice are thrown in the air. Find the probability that the sum of the score is greater than nine or an even number.		
	Ans	$S = ((1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$ $(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$ $(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$ $(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$ $(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$ $(6,1),(6,2),(6,3),(6,4),(6,5),(6,6))$		
		$n(S) = 36$	1	
		$A = ((1,1),(1,3),(1,5),(2,2),(2,4),(2,6),$ $(3,1),(3,3),(3,5),(4,2),(4,4),(4,6),$ $(5,1),(5,3),(5,5),(5,6),(6,2),(6,4),$ $(6,5),(6,6))$	1	
		$n(A) = 20$		
		$p(A) = \frac{n(A)}{n(S)} = \frac{20}{36}$	2	04

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6.		$\therefore\ p(A) = 0.556$		
		--		
	f)	In a test on 2000 electric bulbs , it was found that the life of particular make was normally distributed with average life of 2040 hours and standard deviation of 60 hours. Estimate the no.of bulbs likely to burn for:		
		i) between 1920 hours and 2160 hours		
		ii) more than 2150 hours		
		$\left(\text{Given that : } A(2) = 0.4772, A(1.83) = 0.4664\right)$		
	Ans	Given $\bar{x} = 2040,\ \sigma = 60$		
		Standard normal variate, $Z = \dfrac{x - \bar{x}}{\sigma}$		
		i) For $x = 1920$, $Z = \dfrac{1920 - 2040}{60} = -2$	½	
		For $x = 2160$, $Z = \dfrac{2160 - 2040}{60} = 2$		
		$p = (\text{area between } -2 \text{ and } 2) = A(-2) + A(2)$	½	
		$= 0.4772 + 0.4772$	½	
		$= 0.9544$	½	
		$\therefore$ number of bulbs $= 2000 \times 0.9554 = 1908.8 \approx 1909$		
		ii) For $x = 2150$, $Z = \dfrac{2150 - 2040}{60} = 1.83$	½	
		$p = (\text{area more than } 1.83) = 0.5 - A(1.83)$	½	
		$= 0.5 - 0.4664$		
		$= 0.0336$	½	
		$\therefore$ number of bulbs $= 2000 \times 0.0336 = 67.2 \approx 67$	½	04
		--		
		<u>Important Note</u> *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.*		
		--		