



WINTER- 16 EXAMINATION

Model Answer

Subject Code: 17216

Important Instructions to examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
Q. 1		Attempt any <u>TEN</u> of the following:	20
	a)	If $(a - 2bi) + (b - 3ai) = 5 + 2i$ find a and b	02
	Ans	$(a - 2bi) + (b - 3ai) = 5 + 2i$ $\therefore a - 2bi + b - 3ai = 5 + 2i$ $\therefore (a + b) + (-3a - 2b)i = 5 + 2i$ $\therefore a + b = 5$ $-3a - 2b = 2$ $\therefore 2a + 2b = 10$ $-3a - 2b = 2$ ----- $- a = 12$ $\therefore a = -12$ $b = 17$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	b)	Express in the form $x + iy$, $\frac{(2 + i)^2}{2 + 3i}$, where $x, y \in R$ and $i = \sqrt{-1}$	02
	Ans	$\frac{(2 + i)^2}{2 + 3i}$ $= \frac{4 + 4i + i^2}{2 + 3i}$	$\frac{1}{2}$



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1	b)	$\begin{aligned} &= \frac{4 + 4i - 1}{2 + 3i} = \frac{3 + 4i}{2 + 3i} \\ &= \frac{3 + 4i}{2 + 3i} \times \frac{2 - 3i}{2 - 3i} \\ &= \frac{6 - 9i + 8i - 12i^2}{4 - 9i^2} \\ &= \frac{6 - 9i + 8i + 12}{4 + 9} \\ &= \frac{18 - i}{13} \\ &= \frac{18}{13} - \frac{1}{13}i \end{aligned}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	c)	If $f(x) = x^4 - 2x + 7$ find $f(0) + f(2)$	02
	Ans	$f(0) = 7$ $f(2) = 16 - 4 + 7 = 19$ $\therefore f(0) + f(2) = 7 + 19$ $= 26$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	d)	If $f(x) = 16^x + \log_2 x$, find the value of $f\left(\frac{1}{4}\right)$	02
Ans	$f(x) = 16^x + \log_2 x$, $\therefore f\left(\frac{1}{4}\right) = (16)^{\frac{1}{4}} + \log_2\left(\frac{1}{4}\right)$ $= 2 - \log_2 4$ $= 2 - \log_2 (2)^2$ $= 2 - 2 \log_2 2$ $= 2 - 2(1)$ $= 0$	1	
e)	Evaluate $\lim_{x \rightarrow 1} \left(\frac{x^3 - 1}{x - 1} \right)$	02	
Ans	$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$		



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1	e)	$= \lim_{x \rightarrow 1} \frac{(x-1)(x^2+x+1)}{x-1}$ $= \lim_{x \rightarrow 1} (x^2+x+1)$ $= (1)^2 + 1 + 1$ $= 3$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	f)	<p>Evaluate $\lim_{x \rightarrow 0} \frac{3 \sin x + 4x}{7x - 2 \tan x}$</p>	02
	Ans	$\lim_{x \rightarrow 0} \frac{3 \sin x + 4x}{7x - 2 \tan x}$ $= \lim_{x \rightarrow 0} \frac{\frac{3 \sin x + 4x}{x}}{\frac{7x - 2 \tan x}{x}}$ $= \frac{3 \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) + \frac{4x}{x}}{\frac{7x}{x} - 2 \left(\lim_{x \rightarrow 0} \frac{\tan x}{x} \right)}$ $= \frac{3(1) + 4}{7 - 2(1)}$ $= \frac{7}{5}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	g)	<p>Evaluate $\lim_{x \rightarrow 0} \frac{3^x - 2^x}{\sin \pi x}$</p>	02
	Ans	$\lim_{x \rightarrow 0} \frac{3^x - 2^x}{\sin \pi x}$ $= \lim_{x \rightarrow 0} \frac{\frac{(3^x - 1 - 2^x + 1)}{x}}{\frac{\sin \pi x}{x}}$ $= \frac{\lim_{x \rightarrow 0} \left(\frac{(3^x - 1) - (2^x - 1)}{x} \right)}{\left(\lim_{x \rightarrow 0} \frac{\sin \pi x}{\pi x} \right) \pi}$	$\frac{1}{2}$ $\frac{1}{2}$



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1	g)	$\lim_{x \rightarrow 0} \left(\frac{(3^x - 1)}{x} - \frac{(2^x - 1)}{x} \right)$ $= \frac{\left(\lim_{x \rightarrow 0} \frac{\sin \pi x}{\pi x} \right) \pi}{\left(\lim_{x \rightarrow 0} \frac{3^x - 1}{x} \right) - \left(\lim_{x \rightarrow 0} \frac{2^x - 1}{x} \right)}$ $= \frac{\left(\lim_{x \rightarrow 0} \frac{\sin \pi x}{\pi x} \right) \pi}{\log 3 - \log 2}$ $= \frac{1}{\pi} (\log 3 - \log 2) = \frac{1}{\pi} \log \left(\frac{3}{2} \right)$	<p>½</p> <p>½</p>
	h)	<p>If $y = e^{7x} \cos 7x$, find $\frac{dy}{dx}$</p>	02
	Ans	<p>$y = e^{7x} \cos 7x$</p> $\frac{dy}{dx} = e^{7x} \frac{d}{dx} \cos 7x + \cos 7x \frac{d}{dx} e^{7x}$ $\therefore \frac{dy}{dx} = e^{7x} (-\sin 7x) \cdot 7 + \cos 7x e^{7x} \cdot 7$ $\therefore \frac{dy}{dx} = 7e^{7x} (-\sin 7x + \cos 7x)$	<p>1</p> <p>1</p>
	i)	<p>If $y = \log(x \sin 2x)$, find $\frac{dy}{dx}$</p>	02
Ans	<p>$y = \log(x \sin 2x)$</p> $\frac{dy}{dx} = \frac{1}{x \sin 2x} \frac{d}{dx} (x \sin 2x)$ $\frac{dy}{dx} = \frac{1}{x \sin 2x} (x \cos 2x \cdot 2 + \sin 2x \cdot 1)$ $\frac{dy}{dx} = \frac{(2x \cos 2x + \sin 2x)}{x \sin 2x}$	<p>½</p> <p>1</p> <p>½</p>	
j)	<p>Find $\frac{dy}{dx}$, if $x = 3 \sin 4\theta$, $y = 4 \cos 3\theta$</p>	02	
Ans	<p>$x = 3 \sin 4\theta$, $y = 4 \cos 3\theta$</p>		



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1	j)	$\frac{dx}{d\theta} = 12 \cos 4\theta \quad \text{and} \quad \frac{dy}{d\theta} = -12 \sin 3\theta$ $\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-12 \sin 3\theta}{12 \cos 4\theta}$ $\frac{dy}{dx} = \frac{-\sin 3\theta}{\cos 4\theta}$	<p>½+½</p> <p>½</p> <p>½</p>
	k)	<p>Show that there exist the root of the equation $x^3 - 5x - 11 = 0$ between 2 and 3</p>	02
	Ans	<p>Let $f(x) = x^3 - 5x - 11$</p> <p>$f(2) = -13 < 0$</p> <p>$f(3) = 1 > 0$</p> <p>\therefore root lies between 2 and 3</p>	<p>½</p> <p>½</p> <p>½</p> <p>½</p>
2	l)	<p>Solve the following equations by using Jacobi's method (only first iteration)</p> <p>$4x - y + z = 4, x + 6y + 2z = 9, -x - 2y + 5z = 2$</p>	02
	Ans	<p>Initial approximations : $x_0 = y_0 = z_0 = 0$</p> $x = \frac{4 + y - z}{4}$ $y = \frac{9 - x - 2z}{6}$ $z = \frac{2 + x + 2y}{5}$ <p>$x = 1, \quad y = 1.5, \quad z = 0.4$</p>	<p>½</p> <p>1½</p>
		<p>Attempt any <u>FOUR</u> of the following:</p>	16
	a)	<p>Express $(1 + i)$ in polar form</p>	04
	Ans	<p>Let $z = 1 + i$</p> <p>$\text{Re}(z) = 1, \text{Im}(z) = 1$</p> $r = z = \sqrt{1+1} = \sqrt{2}$ $\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$ <p>Polar form, $z = r(\cos \theta + i \sin \theta)$</p>	<p>1½</p> <p>1½</p>



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2	a)	$\therefore 1 + i = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$	1
	b)	Simplify using De-Moiver's theorem $\frac{(\cos 2\theta + j \sin 2\theta)^{\frac{3}{2}} (\cos \theta - j \sin \theta)^3}{(\cos 3\theta - j \sin 3\theta)^2 (\cos 5\theta - j \sin 5\theta)^{\frac{2}{5}}}$	04
	Ans	$\frac{(\cos 2\theta + j \sin 2\theta)^{\frac{3}{2}} (\cos \theta - j \sin \theta)^3}{(\cos 3\theta - j \sin 3\theta)^2 (\cos 5\theta - j \sin 5\theta)^{\frac{2}{5}}}$ $= \frac{(\cos \theta + j \sin \theta)^3 (\cos \theta + j \sin \theta)^{-3}}{(\cos \theta + j \sin \theta)^{-6} (\cos \theta + j \sin \theta)^{-2}}$ $= (\cos \theta + j \sin \theta)^{3-3+6+2}$ $= (\cos \theta + j \sin \theta)^8$ $= \cos 8\theta + j \sin 8\theta$	2 1 $\frac{1}{2}$ $\frac{1}{2}$
	c)	Use De-Moiver's theorem to solve $x^3 - 1 = 0$	04
	Ans	$x^3 - 1 = 0$ $\therefore x^3 = 1$ <p>Put $x^3 = z$</p> $\therefore x = z^{\frac{1}{3}}$ $\therefore z = 1 + 0i$ $\text{Re}(z) = 1, \text{Im}(z) = 0$ $r = z = \sqrt{1+0} = 1$ $\theta = \tan^{-1} \left(\frac{0}{1} \right) = 0$ $z = r(\cos \theta + i \sin \theta)$ $z = 1(\cos 0 + i \sin 0)$ <p>In general polar form, $z = r(\cos(2\pi k + \theta) + i \sin(2\pi k + \theta))$</p> $z = 1(\cos 2\pi k + i \sin 2\pi k)$ $z^{\frac{1}{3}} = (\cos 2\pi k + i \sin 2\pi k)^{\frac{1}{3}}$ $z = \cos \left(\frac{2\pi k}{3} \right) + i \sin \left(\frac{2\pi k}{3} \right) \quad ; \quad k = 0, 1, 2$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$



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2	e)	$= \frac{x}{1}$ $= x$	1
	f)	<p>If $f(x) = x^2 - 3x + 4$, find x if $f(1-x) = f(2x+1)$</p> <p>Ans</p> $f(1-x) = (1-x)^2 - 3(1-x) + 4$ $= 1 - 2x + x^2 - 3 + 3x + 4$ $= x^2 + x + 2$ $f(2x+1)$ $= (2x+1)^2 - 3(2x+1) + 4$ $= 4x^2 + 4x + 1 - 6x - 3 + 4$ $= 4x^2 - 2x + 2$ <p>Given $f(1-x) = f(2x+1)$</p> $\therefore x^2 + x + 2 = 4x^2 - 2x + 2$ $\therefore -3x^2 + 3x = 0$ $\therefore 3x^2 - 3x = 0$ $3x(x-1) = 0$ $\therefore x = 0, 1$	04
3	a)	<p>If $y = f(x) = \frac{2x-3}{3x-2}$ show that $x = f(y)$</p> <p>Ans</p> $f(y) = \frac{2y-3}{3y-2}$ $= \frac{2\left(\frac{2x-3}{3x-2}\right) - 3}{3\left(\frac{2x-3}{3x-2}\right) - 2}$ $= \frac{2(2x-3) - 3(3x-2)}{3(2x-3) - 2(3x-2)}$ $= \frac{4x - 6 - 9x + 6}{6x - 9 - 6x + 4}$	16
			04



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3	a)	$\begin{aligned} &= \frac{-5x}{-5} \\ &= x \end{aligned}$	$\frac{1}{2}$ $\frac{1}{2}$
	b)	Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$	04
	Ans	$\begin{aligned} &\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3} \\ &= \lim_{x \rightarrow 3} \frac{(x-3)(x+2)}{x^2(x-3) + 1(x-3)} \\ &= \lim_{x \rightarrow 3} \frac{(x-3)(x+2)}{(x^2+1)(x-3)} \\ &= \lim_{x \rightarrow 3} \frac{(x+2)}{(x^2+1)} \\ &= \frac{3+2}{(3)^2+1} \\ &= \frac{5}{10} \\ &= \frac{1}{2} \end{aligned}$	1 1 1 1
c)	Evaluate $\lim_{x \rightarrow 2} \left(\frac{x}{x-2} - \frac{4}{x^2-2x} \right)$	04	
Ans	$\begin{aligned} &\lim_{x \rightarrow 2} \left(\frac{x}{x-2} - \frac{4}{x^2-2x} \right) \\ &= \lim_{x \rightarrow 2} \left(\frac{x}{x-2} - \frac{4}{x(x-2)} \right) \\ &= \lim_{x \rightarrow 2} \left(\frac{x^2}{x(x-2)} - \frac{4}{x(x-2)} \right) \\ &= \lim_{x \rightarrow 2} \frac{x^2-4}{x(x-2)} \\ &= \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x(x-2)} \\ &= \lim_{x \rightarrow 2} \frac{(x+2)}{x} \end{aligned}$	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$	



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3	c)	$= \frac{2+2}{2}$ $= \frac{4}{2}$ $= 2$	$\frac{1}{2}$ 1
	d)	Evaluate $\lim_{\theta \rightarrow 0} \frac{2 \sin \theta - \sin 2\theta}{\theta^3}$	04
	Ans	$\lim_{\theta \rightarrow 0} \frac{2 \sin \theta - \sin 2\theta}{\theta^3}$ $= \lim_{\theta \rightarrow 0} \frac{2 \sin \theta - 2 \sin \theta \cos \theta}{\theta^3}$ $= \lim_{\theta \rightarrow 0} \frac{2 \sin \theta (1 - \cos \theta)}{\theta^3}$ $= \lim_{\theta \rightarrow 0} \frac{2 \sin \theta 2 \sin^2 \left(\frac{\theta}{2}\right)}{\theta^3}$ $= 4 \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \frac{\sin^2 \left(\frac{\theta}{2}\right)}{\theta^2}$ $= 4 \left(\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \right) \left(\lim_{\theta \rightarrow 0} \frac{\sin \left(\frac{\theta}{2}\right)}{\frac{\theta}{2}} \cdot \frac{1}{2} \right)^2$ $= 4 (1) \left(1 \cdot \frac{1}{2} \right)^2$ $= \frac{4}{4}$ $= 1$	$\frac{1}{2}$ 1 1 $\frac{1}{2}$
e)	Evaluate $\lim_{x \rightarrow 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$	04	
Ans	$= \lim_{x \rightarrow 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$ $= \lim_{x \rightarrow 0} \frac{3^x 2^x - 3^x - 2^x + 1}{x^2}$ $= \lim_{x \rightarrow 0} \frac{3^x (2^x - 1) - (2^x - 1)}{x^2}$	1	



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3	e)	$= \lim_{x \rightarrow 0} \frac{(3^x - 1)(2^x - 1)}{x^2}$ $= \left(\lim_{x \rightarrow 0} \frac{3^x - 1}{x} \right) \left(\lim_{x \rightarrow 0} \frac{2^x - 1}{x} \right)$ $= (\log 3)(\log 2)$	1 1 1
	f) Ans	<p>Evaluate $f(x) = \log \left(\frac{1+x}{1-x} \right)$, prove that $f(a) + f(b) = f \left(\frac{a+b}{1+ab} \right)$</p> $f(a) + f(b) = \log \left(\frac{1+a}{1-a} \right) + \log \left(\frac{1+b}{1-b} \right)$ $= \log \left(\frac{1+a}{1-a} \cdot \frac{1+b}{1-b} \right)$ $= \log \left(\frac{1+a+b+ab}{1-a-b+ab} \right)$ $f \left(\frac{a+b}{1+ab} \right) = \log \left(\frac{1 + \frac{a+b}{1+ab}}{1 - \frac{a+b}{1+ab}} \right)$ $= \log \left(\frac{\frac{1+ab+a+b}{1+ab}}{\frac{1+ab-(a+b)}{1+ab}} \right)$ $= \log \left(\frac{1+a+b+ab}{1-a-b+ab} \right)$ <p>$\therefore f(a) + f(b) = f \left(\frac{a+b}{1+ab} \right)$</p> <p>OR</p> $f(a) + f(b) = \log \left(\frac{1+a}{1-a} \right) + \log \left(\frac{1+b}{1-b} \right)$ $= \log \left(\frac{1+a}{1-a} \cdot \frac{1+b}{1-b} \right)$ $= \log \left(\frac{1+a+b+ab}{1-a-b+ab} \right)$ $= \log \left(\frac{1+ab+a+b}{1+ab-(a+b)} \right)$	04 $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$



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3	f)	$= \log \left[\frac{1 + \left(\frac{a+b}{1+ab} \right)}{1 - \left(\frac{a+b}{1+ab} \right)} \right]$ $= f \left(\frac{a+b}{1+ab} \right)$	1 ½
4	a)	<p>Attempt any <u>FOUR</u> of the following:</p> <p>Using first principle of derivative find derivative of $f(x) = \cos x$</p> <p>Ans</p> $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h}$ $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{-2 \sin \left(\frac{x+h+x}{2} \right) \sin \left(\frac{x+h-x}{2} \right)}{h}$ $\frac{dy}{dx} = -2 \lim_{h \rightarrow 0} \frac{\sin \left(\frac{2x+h}{2} \right) \sin \left(\frac{h}{2} \right)}{h}$ $\frac{dy}{dx} = -2 \left(\lim_{h \rightarrow 0} \sin \left(\frac{2x+h}{2} \right) \right) \left(\lim_{h \rightarrow 0} \frac{\sin \left(\frac{h}{2} \right)}{\frac{h}{2}} \cdot \frac{1}{2} \right)$ $\frac{dy}{dx} = -2 (\sin x) \frac{1}{2}$ $\frac{dy}{dx} = -\sin x$	16 04 1 1 1 ½ ½
	b)	<p>If u and v are differentiable functions of x then prove that $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$</p> <p>Ans</p> <p>Given $y = uv$</p> <p>Let $\delta u, \delta v, \delta y$ are small increments in u, v, y respectively corresponding to increment δx in x.</p> <p>$\therefore y + \delta y = (u + \delta u)(v + \delta v)$</p> <p>$y + \delta y = uv + u\delta v + v\delta u + \delta u\delta v$</p> <p>$\delta y = uv + u\delta v + v\delta u + \delta u\delta v - y$</p>	04 ½ ½



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4	b)	$\delta y = uv + u\delta v + v\delta u + \delta u\delta v - uv$ $\delta y = u\delta v + v\delta u + \delta u\delta v$ <p>$\therefore \delta u, \delta v$ are very small.</p> <p>$\therefore \delta u\delta v$ is negligible.</p> <p>$\therefore \delta y = u\delta v + v\delta u$</p> $\therefore \frac{\delta y}{\delta x} = \frac{u\delta v + v\delta u}{\delta x}$ $\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = u \lim_{\delta x \rightarrow 0} \frac{\delta v}{\delta x} + v \lim_{\delta x \rightarrow 0} \frac{\delta u}{\delta x}$ $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>
	c)	<p>If $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$ find $\frac{dy}{dx}$</p> <p>Ans $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$</p> $\therefore \frac{dy}{dx} = \frac{1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \frac{d}{dx} \left(\frac{\cos x}{1 + \sin x}\right)$ $\therefore \frac{dy}{dx} = \frac{1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \left(\frac{(1 + \sin x)(-\sin x) - \cos x(0 + \cos x)}{(1 + \sin x)^2} \right)$ $\therefore \frac{dy}{dx} = \frac{1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \left(\frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2} \right)$ $\therefore \frac{dy}{dx} = \frac{-1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \left(\frac{\sin x + \sin^2 x + \cos^2 x}{(1 + \sin x)^2} \right)$ $\therefore \frac{dy}{dx} = \frac{-1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \left(\frac{\sin x + 1}{(1 + \sin x)^2} \right)$ $\therefore \frac{dy}{dx} = \frac{-1}{\frac{(1 + \sin x)^2 + \cos^2 x}{(1 + \sin x)^2}} \left(\frac{1}{1 + \sin x} \right)$	<p>04</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>



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4	c)	$\therefore \frac{dy}{dx} = \frac{-1}{1 + 2 \sin x + \sin^2 x + \cos^2 x}$	½
		$\frac{dy}{dx} = \frac{-1}{1 + \sin x}$	
		$\therefore \frac{dy}{dx} = \frac{-(1 + \sin x)}{1 + 2 \sin x + 1}$	
		$\therefore \frac{dy}{dx} = \frac{-(1 + \sin x)}{2 + 2 \sin x}$	
		$\therefore \frac{dy}{dx} = \frac{-(1 + \sin x)}{2(1 + \sin x)}$	½
		$\therefore \frac{dy}{dx} = \frac{-1}{2}$	½
		OR	
		$y = \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$	
		$y = \tan^{-1} \left(\frac{\sin \left(\frac{\pi}{2} - x \right)}{1 + \cos \left(\frac{\pi}{2} - x \right)} \right)$	1
		$\therefore y = \tan^{-1} \left(\frac{2 \sin \left(\frac{\pi}{4} - \frac{x}{2} \right) \cos \left(\frac{\pi}{4} - \frac{x}{2} \right)}{2 \cos^2 \left(\frac{\pi}{4} - \frac{x}{2} \right)} \right)$	1
		$\therefore y = \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right)$	1
		$\therefore y = \frac{\pi}{4} - \frac{x}{2}$	½
		$\therefore \frac{dy}{dx} = \frac{-1}{2}$	½
		OR	
$y = \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$			
$\therefore \tan y = \frac{\cos x}{1 + \sin x}$	½		
$\therefore \sec^2 y \frac{dy}{dx} = \frac{(1 + \sin x)(-\sin x) - \cos x(0 + \cos x)}{(1 + \sin x)^2}$	1½		
$\therefore \sec^2 y \frac{dy}{dx} = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2}$	½		



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4		$\therefore \sec^2 y \frac{dy}{dx} = \frac{-(\sin x + \sin^2 x + \cos^2 x)}{(1 + \sin x)^2}$	½	
		$\therefore \sec^2 y \frac{dy}{dx} = \frac{-(\sin x + 1)}{(1 + \sin x)^2}$	½	
		$\therefore \sec^2 y \frac{dy}{dx} = \frac{-1}{1 + \sin x}$		
		$\therefore \frac{dy}{dx} = \frac{-1}{\sec^2 y (1 + \sin x)}$	½	

	d)	If $4x + 3y = \log(4x - 3y)$ find $\frac{dy}{dx}$		04
	Ans	$4x + 3y = \log(4x - 3y)$ $\therefore 4 + 3 \frac{dy}{dx} = \frac{1}{4x - 3y} \frac{d}{dx}(4x - 3y)$	½	
		$\therefore 4 + 3 \frac{dy}{dx} = \frac{1}{4x - 3y} \left(4 - 3 \frac{dy}{dx} \right)$	1	
		$\therefore 4 + 3 \frac{dy}{dx} = \frac{4}{4x - 3y} - \frac{3}{4x - 3y} \frac{dy}{dx}$	½	
		$\therefore 3 \frac{dy}{dx} + \frac{3}{4x - 3y} \frac{dy}{dx} = \frac{4}{4x - 3y} - 4$		
	$\therefore 3 \left(1 + \frac{1}{4x - 3y} \right) \frac{dy}{dx} = 4 \left(\frac{1}{4x - 3y} - 1 \right)$	½		
	$\therefore 3 \left(\frac{4x - 3y + 1}{4x - 3y} \right) \frac{dy}{dx} = 4 \left(\frac{1 - 4x + 3y}{4x - 3y} \right)$	½		
	$\therefore 3 \frac{dy}{dx} = 4 \left(\frac{1 - 4x + 3y}{4x - 3y} \right) \left(\frac{4x - 3y}{4x - 3y + 1} \right)$	½		
	$\therefore \frac{dy}{dx} = \frac{4}{3} \left(\frac{1 - 4x + 3y}{4x - 3y + 1} \right)$	½		

e)	If $x^3 \cdot y^2 = (x + y)^5$ show that $\frac{dy}{dx} = \frac{y}{x}$		04	
Ans	$\log(x^3 y^2) = \log(x + y)^5$ $\log x^3 + \log y^2 = \log(x + y)^5$	½		



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Q. No.	Sub Q. N.	Answer	Marking Scheme
4	e)	$3 \log x + 2 \log y = 5 \log (x + y)$ $3 \frac{1}{x} + 2 \frac{1}{y} \frac{dy}{dx} = 5 \frac{1}{x+y} \left(1 + \frac{dy}{dx} \right)$ $\frac{3}{x} + \frac{2}{y} \frac{dy}{dx} = \frac{5}{x+y} + \left(\frac{5}{x+y} \right) \frac{dy}{dx}$ $\left(\frac{2}{y} - \frac{5}{x+y} \right) \frac{dy}{dx} = \frac{5}{x+y} - \frac{3}{x}$ $\left(\frac{2x + 2y - 5y}{y(x+y)} \right) \frac{dy}{dx} = \frac{5x - 3x - 3y}{x(x+y)}$ $\left(\frac{2x - 3y}{y} \right) \frac{dy}{dx} = \frac{2x - 3y}{x}$ $\therefore \frac{dy}{dx} = \frac{y}{x}$	<p>½</p> <p>1</p> <p>1</p> <p>½</p> <p>½</p>
	f)	<p>If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$</p>	04
	Ans	$x = a(2\theta - \sin 2\theta), y = a(1 - \cos 2\theta)$ $\frac{dx}{d\theta} = a(2 - 2 \cos 2\theta)$ $\therefore \frac{dx}{d\theta} = 2a(1 - \cos 2\theta)$ $\therefore \frac{dy}{d\theta} = 2a \sin 2\theta$ $\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $\therefore \frac{dy}{dx} = \frac{2a \sin 2\theta}{2a(1 - \cos 2\theta)}$ $\therefore \frac{dy}{dx} = \frac{\sin 2\theta}{1 - \cos 2\theta}$ <p>at $\theta = \frac{\pi}{4}$</p> $\therefore \frac{dy}{dx} = \frac{\sin 2\left(\frac{\pi}{4}\right)}{1 - \cos 2\left(\frac{\pi}{4}\right)}$	<p>1</p> <p>1</p> <p>½</p> <p>½</p>



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Q. No.	Sub Q. N.	Answer	Marking Scheme
4	f)	$\therefore \frac{dy}{dx} = \frac{\sin\left(\frac{\pi}{2}\right)}{1 - \cos\left(\frac{\pi}{2}\right)}$ $\therefore \frac{dy}{dx} = \frac{1}{1 - 0}$ $\therefore \frac{dy}{dx} = 1$ <p>OR</p> $x = a(2\theta - \sin 2\theta), y = a(1 - \cos 2\theta)$ $\frac{dx}{d\theta} = a(2 - 2\cos 2\theta)$ $\therefore \frac{dx}{d\theta} = 2a(1 - \cos 2\theta)$ $\therefore \frac{dy}{d\theta} = 2a \sin 2\theta$ $\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $\therefore \frac{dy}{dx} = \frac{2a \sin 2\theta}{2a(1 - \cos 2\theta)}$ $\therefore \frac{dy}{dx} = \frac{\sin 2\theta}{1 - \cos 2\theta}$ $\therefore \frac{dy}{dx} = \frac{2 \sin \theta \cos \theta}{2 \sin^2 \theta}$ $\therefore \frac{dy}{dx} = \frac{\cos \theta}{\sin \theta}$ $\therefore \frac{dy}{dx} = \cot \theta$ <p>at $\theta = \frac{\pi}{4}$</p> $\therefore \frac{dy}{dx} = \cot \frac{\pi}{4}$ $\therefore \frac{dy}{dx} = 1$	<p>1</p> <p>1</p> <p>1</p> <p>½</p> <p>½</p> <p>½</p>



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Q. No.	Sub Q. N.	Answer	Marking Scheme
5		Attempt any <u>FOUR</u> of the following:	16
	a)	Evaluate $\lim_{x \rightarrow 0} \frac{(5^x - 1) \tan x}{\sqrt{x^2 + 16} - 4}$	04
	Ans	$\lim_{x \rightarrow 0} \frac{(5^x - 1) \tan x}{\sqrt{x^2 + 16} - 4}$ $= \lim_{x \rightarrow 0} \frac{(5^x - 1) \tan x}{\sqrt{x^2 + 16} - 4} \times \frac{\sqrt{x^2 + 16} + 4}{\sqrt{x^2 + 16} + 4}$ $= \lim_{x \rightarrow 0} \frac{(5^x - 1) \tan x (\sqrt{x^2 + 16} + 4)}{x^2 + 16 - 16}$ $= \lim_{x \rightarrow 0} \frac{(5^x - 1) \tan x (\sqrt{x^2 + 16} + 4)}{x^2}$ $= \left(\lim_{x \rightarrow 0} \frac{5^x - 1}{x} \right) \left(\lim_{x \rightarrow 0} \frac{\tan x}{x} \right) \left(\lim_{x \rightarrow 0} \sqrt{x^2 + 16} + 4 \right)$ $= (\log 5)(1)(\sqrt{16} + 4)$ $= (\log 5)(1)(4 + 4)$ $= 8 \log 5$	1 1 1 1
	b)	Evaluate: $\lim_{x \rightarrow 3} \frac{\log x - \log 3}{x - 3}$	04
	Ans	$\lim_{x \rightarrow 3} \frac{\log x - \log 3}{x - 3}$ <p>Put $x = 3 + h$ as $x \rightarrow 3, h \rightarrow 0$</p> $= \lim_{h \rightarrow 0} \frac{\log(3 + h) - \log 3}{3 + h - 3}$ $= \lim_{h \rightarrow 0} \frac{\log \left(\frac{3 + h}{3} \right)}{h}$ $= \lim_{h \rightarrow 0} \frac{1}{h} \log \left(1 + \frac{h}{3} \right)$ $= \lim_{h \rightarrow 0} \log \left(1 + \frac{h}{3} \right)^{\frac{1}{h}}$ $= \log \left[\lim_{h \rightarrow 0} \left(1 + \frac{h}{3} \right)^{\frac{3}{h}} \right]^{\frac{1}{3}}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$



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Q. No.	Sub Q. N.	Answer	Marking Scheme																				
5	b)	$= \log e^{\frac{1}{3}}$ $= \frac{1}{3} \log e$ $= \frac{1}{3}$	½																				
	c)	Using Bisection method find the approximate root of the equation $x^3 - 5x + 1 = 0$ (three iterations only)	04																				
	Ans	Let $f(x) = x^3 - 5x + 1$ $f(2) = -1 < 0$ $f(3) = 13 > 0$ \therefore root lies in (2,3) $x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$ $f(x_1) = 4.125 > 0$ the root lies in (2, 2.5) $x_2 = \frac{a+x_1}{2} = \frac{2+2.5}{2} = 2.25$ $f(x_2) = 1.14 > 0$ the root lies in (2, 2.25) $x_3 = \frac{a+x_2}{2} = \frac{2+2.25}{2} = 2.125$ OR Let $f(x) = x^3 - 5x + 1$ $f(2) = -1 < 0$ $f(3) = 13 > 0$ \therefore root lies in (2,3)	½ ½ ½ ½ ½ ½ ½ ½ ½ ½ ½ ½ ½ ½																				
		<table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Iterations</th> <th style="padding: 5px;">a</th> <th style="padding: 5px;">b</th> <th style="padding: 5px;">$x = \frac{a+b}{2}$</th> <th style="padding: 5px;">$f(x)$</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">I</td> <td style="text-align: center;">2</td> <td style="text-align: center;">3</td> <td style="text-align: center;">2.5</td> <td style="text-align: center;">4.125</td> </tr> <tr> <td style="text-align: center;">II</td> <td style="text-align: center;">2</td> <td style="text-align: center;">2.5</td> <td style="text-align: center;">2.25</td> <td style="text-align: center;">1.14</td> </tr> <tr> <td style="text-align: center;">III</td> <td style="text-align: center;">2</td> <td style="text-align: center;">2.25</td> <td style="text-align: center;">2.125</td> <td style="text-align: center;">---</td> </tr> </tbody> </table>	Iterations	a	b	$x = \frac{a+b}{2}$	$f(x)$	I	2	3	2.5	4.125	II	2	2.5	2.25	1.14	III	2	2.25	2.125	---	1 1 ½
Iterations	a	b	$x = \frac{a+b}{2}$	$f(x)$																			
I	2	3	2.5	4.125																			
II	2	2.5	2.25	1.14																			
III	2	2.25	2.125	---																			



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Q. No.	Sub Q. N.	Answer	Marking Scheme																												
5	d) Ans	Find the root of $x^2 - x - 1 = 0$ by using Regula Falsi method up to third approximation.	04																												
		Let $f(x) = x^2 - x - 1$																													
		$f(1) = -1 < 0$	½																												
		$f(2) = 1 > 0$	½																												
		∴ the root lies in (1, 2)	½																												
		$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(1) - 2(-1)}{1 + 1} = 1.5$	½																												
		$f(x_1) = -0.25 < 0$	½																												
		the root lies in (1.5, 2)																													
		$x_2 = \frac{1.5(1) - 2(-0.25)}{1 + 0.25} = 1.6$	½																												
		$f(x_2) = -0.04 < 0$	½																												
		the root lies in (1.6, 2)																													
		$x_3 = \frac{1.6(1) - 2(-0.04)}{1 + 0.04} = 1.615$	½																												
		<i>OR</i>																													
		Let $f(x) = x^2 - x - 1$	½																												
$f(1) = -1 < 0$	½																														
$f(2) = 1 > 0$	½																														
∴ the root lies in (1, 2)	½																														
		<table border="1"><thead><tr><th>Iterations</th><th>a</th><th>b</th><th>f(a)</th><th>f(b)</th><th>$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$</th><th>f(x)</th></tr></thead><tbody><tr><td>I</td><td>1</td><td>2</td><td>-1</td><td>1</td><td>1.5</td><td>-0.25</td></tr><tr><td>II</td><td>1.5</td><td>2</td><td>-0.25</td><td>1</td><td>1.6</td><td>-0.04</td></tr><tr><td>III</td><td>1.6</td><td>2</td><td>-0.04</td><td>1</td><td>1.615</td><td>---</td></tr></tbody></table>	Iterations	a	b	f(a)	f(b)	$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$	f(x)	I	1	2	-1	1	1.5	-0.25	II	1.5	2	-0.25	1	1.6	-0.04	III	1.6	2	-0.04	1	1.615	---	1 1 ½
Iterations	a	b	f(a)	f(b)	$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$	f(x)																									
I	1	2	-1	1	1.5	-0.25																									
II	1.5	2	-0.25	1	1.6	-0.04																									
III	1.6	2	-0.04	1	1.615	---																									
e) Ans	Find the positive root of $x^3 + x - 1 = 0$ by Newton-Raphson method up to three iterations only	04																													
Let $f(x) = x^3 + x - 1$																															
$f(0) = -1 < 0$		½																													
$f(1) = 1 > 0$		½																													
$f'(x) = 3x^2 + 1$		½																													



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Q. No.	Sub Q. N.	Answer	Marking Scheme
5	e)	<p>Initial root $x_0 = 1$</p> <p>$\therefore f'(1) = 4$</p> $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(1)}{f'(1)} = 0.75$ $x_2 = 0.75 - \frac{f(0.75)}{f'(0.75)} = 0.686$ $x_3 = 0.686 - \frac{f(0.686)}{f'(0.686)} = 0.682$ <p>OR</p> <p>Let $f(x) = x^3 + x - 1$</p> <p>$f(0) = -1 < 0$</p> <p>$f(1) = 1 > 0$</p> <p>$f'(x) = 3x^2 + 1$</p> <p>Initial root $x_0 = 1$</p> $x_i = x - \frac{f(x)}{f'(x)}$ $x_i = x - \frac{x^3 + x - 1}{3x^2 + 1}$ $x_i = \frac{3x^3 + x - x^3 - x + 1}{3x^2 + 1}$ $x_i = \frac{2x^3 + 1}{3x^2 + 1}$ <p>$\therefore f'(1) = 4$</p> <p>$x_1 = 0.75$</p> <p>$x_2 = 0.686$</p> <p>$x_3 = 0.682$</p>	<p>1</p> <p>1</p> <p>½</p> <p>½</p> <p>½</p> <p>½</p> <p>1</p> <p>½</p> <p>½</p> <p>½</p>
	f)	<p>Use Newton-Raphson method to find $\sqrt[3]{20}$ correct to three decimal places. (third iteration)</p> <p>Ans Let $x = \sqrt[3]{20}$</p> <p>$\therefore x^3 = 20$</p> <p>$\therefore x^3 - 20 = 0$</p> <p>$\therefore f(x) = x^3 - 20$</p> <p>$f(2) = -12 < 0$</p>	<p>04</p> <p>½</p>



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Q. No.	Sub Q. N.	Answer	Marking Scheme
5	f)	$f(3) = 7 > 0$	½
		$f'(x) = 3x^2$	½
		Initial root $x_0 = 3$	
		$\therefore f'(3) = 27$	
		$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{f(3)}{f'(3)} = 2.741$	1
		$x_2 = 2.74 - \frac{f(2.741)}{f'(2.741)} = 2.715$	1
		$x_2 = 2.71 - \frac{f(2.715)}{f'(2.715)} = 2.714$	½
		OR	
		Let $x = \sqrt[3]{20}$	
		$\therefore x^3 = 20$	
$\therefore x^3 - 20 = 0$			
$\therefore f(x) = x^3 - 20$			
$f(2) = -12 < 0$	½		
$f(3) = 7 > 0$	½		
$f'(x) = 3x^2$	½		
Initial root $x_0 = 3$			
$\therefore f'(3) = 27$			
$x_i = x - \frac{f(x)}{f'(x)} = x - \frac{x^3 - 20}{3x^2}$			
$= \frac{3x^3 - x^3 + 20}{3x^2}$			
$= \frac{2x^3 + 20}{3x^2}$	1		
$x_1 = 2.741$	½		
$x_2 = 2.715$	½		
$x_3 = 2.714$	½		
6	a)	<p>Attempt any FOUR of the following:</p> <p>If $y = (x + \sqrt{x^2 + 1})^m$ show that $(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - m^2 y = 0$</p>	16
			04



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Q. No.	Sub Q. N.	Answer	Marking Scheme
6	Ans	$y = (x + \sqrt{x^2 + 1})^m$	
		$\therefore \frac{dy}{dx} = m (x + \sqrt{x^2 + 1})^{m-1} \frac{d}{dx} (x + \sqrt{x^2 + 1})$	½
		$\therefore \frac{dy}{dx} = m (x + \sqrt{x^2 + 1})^{m-1} \left(1 + \frac{1}{2\sqrt{x^2 + 1}} 2x \right)$	½
		$\therefore \frac{dy}{dx} = m (x + \sqrt{x^2 + 1})^{m-1} \left(1 + \frac{x}{\sqrt{x^2 + 1}} \right)$	
		$\therefore \frac{dy}{dx} = m (x + \sqrt{x^2 + 1})^{m-1} \left(\frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}} \right)$	
		$\therefore \frac{dy}{dx} = m (x + \sqrt{x^2 + 1})^m \frac{1}{\sqrt{x^2 + 1}}$	1
		$\therefore \frac{dy}{dx} = m y \frac{1}{\sqrt{x^2 + 1}}$	
		$\therefore \sqrt{x^2 + 1} \frac{dy}{dx} = m y$	
		$\therefore (x^2 + 1) \left(\frac{dy}{dx} \right)^2 = m^2 y^2$	½
		$\therefore (x^2 + 1) 2 \frac{dy}{dx} \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 2x = 2m^2 y \frac{dy}{dx}$	1
		$\therefore (x^2 + 1) \frac{d^2 y}{dx^2} + \frac{dy}{dx} x = m^2 y$	½
		$\therefore (x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - m^2 y = 0$	

	b)	If $x = 2 \cos t - \cos 2t, y = 2 \sin t - \sin 2t$ find $\frac{d^2 y}{dx^2}$ at $t = \frac{\pi}{2}$	04
	Ans	$x = 2 \cos t - \cos 2t, y = 2 \sin t - \sin 2t$	
		$\frac{dx}{dt} = -2 \sin t + 2 \sin 2t$	½
		$\frac{dy}{dt} = 2 \cos t - 2 \cos 2t$	½
		$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	



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Q. No.	Sub Q. N.	Answer	Marking Scheme
6	b)	$\therefore \frac{dy}{dx} = \frac{2 \cos t - 2 \cos 2t}{-2 \sin t + 2 \sin 2t}$ $\therefore \frac{dy}{dx} = \frac{\cos t - \cos 2t}{-\sin t + \sin 2t}$ $\therefore \frac{d^2 y}{dx^2} = \frac{(-\sin t + \sin 2t)(-\sin t + 2 \sin 2t) - (\cos t - \cos 2t)(-\cos t + 2 \cos 2t)}{(-\sin t + \sin 2t)^2} \frac{dt}{dx}$ $\therefore \frac{d^2 y}{dx^2} = \frac{(-\sin t + \sin 2t)(-\sin t + 2 \sin 2t) - (\cos t - \cos 2t)(-\cos t + 2 \cos 2t)}{(-\sin t + \sin 2t)^2} \frac{1}{(-2 \sin t + 2 \sin 2t)}$ $\therefore \frac{d^2 y}{dx^2} = \frac{(-\sin t + \sin 2t)(-\sin t + 2 \sin 2t) - (\cos t - \cos 2t)(-\cos t + 2 \cos 2t)}{2(-\sin t + \sin 2t)^3}$ $\therefore \frac{d^2 y}{dx^2} = \frac{\sin^2 t + 2 \sin^2 2t - 3 \sin t \sin 2t + \cos^2 t + 2 \cos^2 2t - 3 \cos t \cos 2t}{2(-\sin t + \sin 2t)^3}$ <p>at $t = \frac{\pi}{2}$</p> $\therefore \frac{d^2 y}{dx^2} = \frac{\sin^2\left(\frac{\pi}{2}\right) + 2 \sin^2 2\left(\frac{\pi}{2}\right) - 3 \sin\left(\frac{\pi}{2}\right) \sin 2\left(\frac{\pi}{2}\right) + \cos^2\left(\frac{\pi}{2}\right) + 2 \cos^2 2\left(\frac{\pi}{2}\right) - 3 \cos\left(\frac{\pi}{2}\right) \cos 2\left(\frac{\pi}{2}\right)}{2\left(-\sin\left(\frac{\pi}{2}\right) + \sin 2\left(\frac{\pi}{2}\right)\right)^3}$ $\therefore \frac{d^2 y}{dx^2} = \frac{(1)^2 + 0 - 0 + 0 + 2(-1)^2 - 0}{2(-1+0)^3}$ $\therefore \frac{d^2 y}{dx^2} = \frac{3}{2(-1)}$ $\therefore \frac{d^2 y}{dx^2} = \frac{3}{-2}$	<p>1/2</p> <p>1</p> <p>1/2</p>
		<p>c) Solve the following equations by Gauss elimination method</p> $x + 2y + 3z = 14 \quad , \quad 3x + 3y + 5z = 24 \quad , \quad 4x + 5y + 7z = 35$ <p>Ans</p> $x + 2y + 3z = 14$ $3x + 3y + 5z = 24$ $4x + 5y + 7z = 35$ $3x + 6y + 9z = 42 \qquad \qquad \qquad 4x + 8y + 12z = 56$ $3x + 3y + 5z = 24 \qquad \qquad \text{and} \qquad \qquad 4x + 5y + 7z = 35$ $- \text{-----} \qquad \qquad \qquad - \text{-----}$ $3y + 4z = 18 \qquad \qquad \qquad 3y + 5z = 21$	<p>1</p> <p>04</p> <p>1/2 + 1/2</p>



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6	c)	$3y + 4z = 18$ $3y + 5z = 21$ <p>-----</p> $-z = -3$ $\therefore z = 3$ $\therefore x = 1$ $y = 2$ $z = 3$ <p><i>Note: In the above solution, first x is eliminated and then y is eliminated to find the value of z first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking.</i></p>	1 1 1
	d)	<p>Solve the following equations by Jacobi's method (take three iterations)</p> $5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20$ <p>Ans</p> $x = \frac{1}{5}(12 - 2y - z)$ $y = \frac{1}{4}(15 - x - 2z)$ $z = \frac{1}{5}(20 - x - 2y)$ <p>Starting with $x_0 = y_0 = z_0 = 0$</p> $x_1 = 2.4$ $y_1 = 3.75$ $z_1 = 4$ $x_2 = 0.1$ $y_2 = 1.15$ $z_2 = 2.02$ $x_3 = 1.536$ $y_3 = 2.715$ $z_3 = 3.52$	04 1 1 1



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Q. No.	Sub Q. N.	Answer	Marking Scheme
		<p><u>Important Note</u></p> <p><i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p> <p>-----</p> <p>-----</p>	