SUMMER – 17 EXAMINATION

Model Answer

Subject Code: 17301

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1		**Attempt any TEN of the following:**	**20**
	a)	Find the point on the curve $y = x^2 - 6x + 8$ where the tangent is parallel to X-axis	**02**
	Ans	$y = x^2 - 6x + 8$	
		$\therefore \frac{dy}{dx} = 2x - 6$	½
		$\because$ tangent is parallel to X-axis	½
		$\therefore \frac{dy}{dx} = 0$	½
		$\therefore 2x - 6 = 0$	
		$\therefore x = 3$	
		$y = 3^2 - 6(3) + 8 = -1$	½
		$\therefore$ Point is $(3, -1)$	
		--	
	b)	Find the radius of curvature of the curve $xy = c$ at point (c, c)	**02**
	Ans	$xy = c$	
		$x\frac{dy}{dx} + y = 0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	b)	$\therefore \frac{dy}{dx} = -\frac{y}{x}$	
		$\frac{d^2y}{dx^2} = -\left(\frac{x\frac{dy}{dx} - y}{x^2}\right)$	½
		$at\left(c, c\right)$	
		$\frac{dy}{dx} = -\frac{c}{c} = -1$	
		$\frac{d^2y}{dx^2} = -\left(\frac{c\left(-1\right) - c}{c^2}\right) = \frac{2}{c}$	½
		$\therefore \text{Radius of curvature} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \left(-1\right)^2\right]^{3/2}}{\frac{2}{c}}$	½
		$= \frac{c}{2}\left(2\sqrt{2}\right) = \sqrt{2}\ c$	½
		--	
	c)	$\text{Evaluate} \int \frac{1}{\sin^{-1} x\sqrt{1 - x^2}} dx$	**02**
	Ans	$\int \frac{1}{\sin^{-1} x\sqrt{1 - x^2}} dx$	
		$\text{put } \sin^{-1} x = t$	
		$\therefore \frac{1}{\sqrt{1 - x^2}} dx = dt$	½
		$= \int \frac{1}{t}\ dt$	½
		$= \log t + c$	½
		$= \log\left(\sin^{-1} x\right) + c$	½
		--	
	d)	$\text{Evaluate} \int \frac{1}{\sqrt[4]{\left(2 - 3x\right)^3}} dx$	**02**
	Ans	$\int \frac{1}{\sqrt[4]{\left(2 - 3x\right)^3}} dx$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	d)	$= \int \frac{1}{(2-3x)^{\frac{3}{4}}} dx$	½
		$= \int (2-3x)^{-\frac{3}{4}} dx$	
		$= \frac{(2-3x)^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} \cdot \frac{-1}{3} = \frac{(2-3x)^{\frac{1}{4}}}{\frac{1}{4}} \cdot \frac{-1}{3} + c$	1
		$= -\frac{4}{3}(2-3x)^{\frac{1}{4}} + c$	½
		--	
	e)	Evaluate $\int \tan^{-1} x dx$	**02**
	Ans	$\int \tan^{-1} x.1dx$	
		$= \tan^{-1} x \int 1dx - \int \left(\int 1dx. \frac{d\left(\tan^{-1} x\right)}{dx} \right) dx$	½
		$= \tan^{-1} x.x - \int \frac{x}{1+x^2} dx$	½
		$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$	½
		$= x \tan^{-1} x - \frac{1}{2} \log\left(1+x^2\right) + c$	½
		--	
	f)	Evaluate $\int_0^{\pi/2} \sin^3 x dx$	**02**
	Ans	$\int_0^{\pi/2} \sin^3 x dx$	
		$= \int_0^{\pi/2} \sin^2 x. \sin x dx$	
		$= \int_0^{\pi/2} \left(1-\cos^2 x\right) \sin x dx$	½
		Put $\cos x = t$ when $x = 0$, $t = 1$	
		$\therefore -\sin x dx = dt$ when $x = \frac{\pi}{2}$, $t = 0$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	f)	$= \int_1^0 \left(1 - t^2\right) - dt$	
		$= \int_1^0 \left(-1 + t^2\right) dt$	
		$= \left[-t + \frac{t^3}{3}\right]_1^0$	½
		$= (0) - \left(-1 + \frac{1}{3}\right)$	
		$= \frac{2}{3}$	½
		OR	
		$\int_0^{\pi/2} \sin^3 dx$	
		$= \int_0^{\pi/2} \frac{3\sin x - \sin 3x}{4} dx$	½
		$= \frac{1}{4}\left[3(-\cos x) + \frac{\cos 3x}{3}\right]_0^{\pi/2}$	½
		$= \frac{1}{4}\left[0 + 0 + 3 - \frac{1}{3}\right]$	½
		$= \frac{1}{4} \cdot \frac{8}{3}$	
		$= \frac{2}{3}$	½
		---	
	g)	Find the area of the region bounded by $x^2 = 16y,\ y = 1,\ y = 4$ and Y-axis in first quadrant	**02**
	Ans	$A = \int_a^b x\,dy$	
		$= \int_1^4 4\sqrt{y}\ dy$	½
		$= 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	g)	$=\frac{8}{3}\left(4^{\frac{3}{2}}-1^{\frac{3}{2}}\right)$	½
		$=\frac{8}{3}(7)$	
		$=\frac{56}{3}$	½
		--	
	h)	Determine the order and degree of $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=my$	**02**
	Ans	Order=2	1
		Degree=1	1
		--	
	i)	Form the differential equation if $y=4(x-A)^2$. Where A is arbitrary constant	**02**
	Ans	$y=4(x-A)^2$	
		$\therefore \frac{dy}{dx}=8(x-A)$	½
		$y=4\left(\frac{1}{8}\frac{dy}{dx}\right)^2$	½
		$y=4\frac{1}{64}\left(\frac{dy}{dx}\right)^2$	½
		$\therefore \left(\frac{dy}{dx}\right)^2=16y$	½
		--	
	j)	A fair die is rolled. What is the probability that the number on the die is a prime number	**02**
	Ans	$S=\{1,2,3,4,5,6\}$	
		$\therefore \text{n}(s)=6$	
		$A=\{2,3,5\}$	½
		$\therefore \text{n}(A)=3$	
		$p(A)=\frac{n(A)}{n(s)}=\frac{3}{6}$	½
		$=\frac{1}{2}$ or 0.5	1
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	k)	From 4 men and 2 women, 3 persons are chosen at random to form a committee. Find the probability that the committee consists of at least one person of either sex.	**02**
	Ans	$n(S) = {}^6c_3 = 20$	½
		$n(A) = {}^4c_1 \times {}^2c_2 + {}^4c_2 \times {}^2c_1$	1
		$= 16$	
		$p(A) = \frac{16}{20} = \frac{4}{5}$ or 0.8	½
		--	
	l)	A person fires 10 shorts at target. The probability that any shot will hit the target is $\frac{3}{5}$.Find the probability that the target is hit exactly 5 times.	**02**
	Ans	Given : $n = 10,\ p = \frac{3}{5},\ q = 1 - p = 1 - \frac{3}{5} = \frac{2}{5}$	
		$p(r) = {}^nc_r\, p^r q^{n-r}$	
		$p(5) = {}^{10}c_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^5$	1
		$= 0.2007$	1
		--	
	m)	Evaluate $\int \frac{1}{\sqrt{9-4x^2}}dx$	**02**
	Ans.	$\int \frac{1}{\sqrt{9-4x^2}}dx$ OR $\int \frac{1}{2\sqrt{\frac{9}{4}-x^2}}dx$	
		$= \int \frac{1}{\sqrt{3^2-(2x)^2}}dx$ OR $= \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{3}{2}\right)^2 - x^2}}dx$	1
		$= \sin^{-1}\left(\frac{2x}{3}\right).\frac{1}{2} + c$ OR $= \frac{1}{2}\sin^{-1}\left(\frac{x}{3/2}\right) + c = \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + c$	1
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1	n)	Evaluate $\int_1^e \frac{1}{x}.\log x dx$	**02**
	Ans	$\int_1^e \frac{1}{x}.\log x dx$	
		Put $\log x = t$ when $x = 1$ $t = \log 1 = 0$	
		$\frac{1}{x}dx = dt$ when $x = e$ $t = \log e = 1$	½
		$= \int_0^1 t dt$	½
		$= \left[\frac{t^2}{2}\right]_0^1$	½
		$= \frac{1}{2}$	½
		--	
2		**Attempt any <u>FOUR</u> of the following:**	**16**
	a)	Evaluate $\int \tan^{\varsigma} x \, dx$	**04**
		Considering index as 3	
		$\therefore \int \tan^3 x dx$	
		$= \int \tan^2 x \tan x dx$	½
		$= \int \left(\sec^2 x - 1\right) \tan x dx$	
		$= \int \left(\tan x \sec^2 x - \tan x\right) dx$	
		$= \int \tan x \sec^2 x dx - \int \tan x dx$	
		In first integral, put $\tan x = t$	1
		$\therefore \sec^2 x dx = dt$	1
		$= \int t dt - \log\left(\sec x\right) + c$	1
		$= \frac{t^2}{2} - \log\left(\sec x\right) + c$	½
		$= \frac{\tan^2 x}{2} - \log\left(\sec x\right) + c$	
		<u>Note:</u> If student attempted to solve the problem assuming any index value then consider it and reward appropriate marks to it.	
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Q. No.	Sub Q. N.	Answer	Marking Scheme
2	b)	Evaluate $\int \frac{\log x}{x(2+\log x)(3+\log x)}dx$	**04**
	Ans	$\int \frac{\log x}{x(2+\log x)(3+\log x)}dx$	
		Put $\log x = t$	
		$\therefore \frac{1}{x}dx = dt$	½
		$\int \frac{t}{(2+t)(3+t)}dt$	½
		consider $\frac{t}{(2+t)(3+t)} = \frac{A}{2+t} + \frac{B}{3+t}$	½
		$\therefore t = A(3+t) + B(2+t)$	
		Put $t = -2$	
		$A = -2$	½
		Put $t = -3$	
		$B = 3$	½
		$\therefore \frac{t}{(2+t)(3+t)} = \frac{-2}{2+t} + \frac{3}{3+t}$	½
		$\therefore \int \frac{t}{(2+t)(3+t)}dt = \int \frac{-2}{2+t} + \frac{3}{3+t}dt$	
		$= -2\log(2+t) + 3\log(3+t) + c$	½
		$= -2\log(2+\log x) + 3\log(3+\log x) + c$	½
		---	
	c)	Find the equations of the tangent and normal to the ellipse $2x^2 + 3y^2 = 5$ which is perpendicular to the line $3x + 2y + 7 = 0$	**04**
	Ans	Slope of line $3x + 2y + 7 = 0$ is	
		$m_1 = \frac{-3}{2}$	
		$\because 2x^2 + 3y^2 = 5$	
		$\therefore 4(x) + 6y\frac{dy}{dx} = 0$	
		$\therefore \frac{dy}{dx} = \frac{-4x}{6y} = \frac{-2x}{3y}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	c)	$\therefore$ slope of tangent $= m_2 = \dfrac{-2x}{3y}$	½
		Line and tangent are perpendicular	
		$\therefore m_1.m_2 = -1$	
		$\therefore \dfrac{-3}{2}.\dfrac{-2x}{3y} = -1$	½
		$\therefore x = -y \quad \therefore y = -x$	
		$\therefore 2x^2 + 3(-x)^2 = 5$	
		$2x^2 + 3x^2 = 5$	
		$\therefore x^2 = 1$	
		$\therefore x = \pm 1$	
		if $x = 1 \quad y = -1 \quad \therefore$ point is $(1,-1)$	½
		if $x = -1 \quad y = 1 \quad \therefore$ point is $(-1,1)$	½
		Equation of tangent at $(1,-1)$ is	
		$y + 1 = \dfrac{2}{3}(x-1)$	½
		$\therefore 2x - 3y - 5 = 0$	
		Equation of tangent at $(-1,1)$ is	
		$y - 1 = \dfrac{2}{3}(x+1)$	
		$\therefore 2x - 3y + 5 = 0$	½
		Equation of normal at $(1,-1)$	
		$y + 1 = \dfrac{-3}{2}(x-1)$	
		$\therefore 3x + 2y - 1 = 0$	½
		Equation of normal at $(-1,1)$ is	
		$y - 1 = \dfrac{-3}{2}(x+1)$	
		$\therefore 3x + 2y + 1 = 0$	½
		--	
	d)	Find the radius of curvature for the curve $x = a\cos^3\theta,\ y = a\sin^3\theta\ at\ \theta = \dfrac{\pi}{4}$	**04**
	Ans	$x = a\cos^3\theta \qquad y = a\sin^3\theta$	
		$\dfrac{dx}{d\theta} = -3a\cos^2\theta\sin\theta \qquad \dfrac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$	½+½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
2	d)	$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$	
		$\frac{dy}{dx}=-\tan\theta$	½
		$\frac{d^2y}{dx^2}=-\sec^2\theta\frac{d\theta}{dx}$	
		$=-\sec^2\theta\frac{1}{\frac{dx}{d\theta}}$	
		$=-\sec^2\theta\frac{1}{-3a\cos^2\theta\sin\theta}$	½
		at $\theta=\frac{\pi}{4}$	
		$\frac{dy}{dx}=-\tan\left(\frac{\pi}{4}\right)=-1$	½
		$\frac{d^2y}{dx^2}=-\sec^2\left(\frac{\pi}{4}\right)\frac{1}{-3a\cos^2\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)}$	
		$=-\left(\sqrt{2}\right)^2.\frac{1}{-3a\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)}$	
		$=\frac{2}{3a\frac{1}{2\sqrt{2}}}$	
		$=\frac{4\sqrt{2}}{3a}$	½
		Radius of curvature $=\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$	
		$=\frac{\left(1+(-1)^2\right)^{3/2}}{\frac{4\sqrt{2}}{3a}}$	½
		$=\frac{3a}{2}$ or $(1.5)a$	½
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	e)	A bullet is fired into a mud tank and penetrates $(120t - 3600t^2)$ meters in 't' seconds after impact. Calculate maximum depth of penetration.	04
	Ans	$y = 120t - 3600t^2$	
		$\frac{dy}{dt} = 120 - 7200t$	½
		$\frac{d^2y}{dt^2} = -7200 < 0 \ \therefore$ Depth is maximum.	½
		Let $\frac{dy}{dt} = 0$	½
		$\therefore 120 - 7200t = 0$	
		$\therefore t = \frac{1}{60}$	1
		$\therefore$ Maximum depth $y = 120\left(\frac{1}{60}\right) - 3600\left(\frac{1}{60}\right)^2$	½
		$= 1$ meter	1
		--	
	f)	Evaluate $\int \frac{5x-4}{x^2-8x+12}dx$	04
	Ans	$\int \frac{5x-4}{x^2-8x+12}dx$	
		Consider $\frac{5x-4}{x^2-8x+12} = \frac{5x-4}{(x-6)(x-2)}$	½
		$\frac{5x-4}{(x-6)(x-2)} = \frac{A}{x-6} + \frac{B}{x-2}$	½
		$\therefore 5x - 4 = A(x-2) + B(x-6)$	
		Put $x = 6$	
		$\therefore A = \frac{26}{4} = \frac{13}{2}$	½
		Put $x = 2$	
		$\therefore B = \frac{-6}{4} = \frac{-3}{2}$	½
		$\frac{5x-4}{x^2-8x+12} = \frac{\frac{13}{2}}{x-6} + \frac{\frac{-3}{2}}{x-2}$	½
		$\therefore \int \frac{5x-4}{x^2-8x+12}dx = \int \left(\frac{\frac{13}{2}}{x-6} + \frac{\frac{-3}{2}}{x-2} \right) dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	f)	$I=\frac{13}{2}\log(x-6)-\frac{3}{2}\log(x-2)+c$	1
		--	
3		**Attempt any FOUR of the following:**	**16**
	a)	Evaluate $\int_{-3/2}^{1/2}\frac{1}{4x^2+12x+13}\,dx$	**04**
	Ans	$\int_{-3/2}^{1/2}\frac{1}{4x^2+12x+13}\,dx$	
		$=\frac{1}{4}\int_{-3/2}^{1/2}\frac{1}{x^2+3x+\frac{13}{4}}\,dx$	½
		Third term $=\frac{(3x)^2}{4.(x)^2}=\frac{9}{4}$	½
		$=\frac{1}{4}\int_{-3/2}^{1/2}\frac{1}{x^2+3x+\frac{9}{4}-\frac{9}{4}+\frac{13}{4}}\,dx$	½
		$=\frac{1}{4}\int_{-3/2}^{1/2}\frac{1}{\left(x+\frac{3}{2}\right)^2+1^2}\,dx$	½
		$=\left[\frac{1}{4}\tan^{-1}\left(x+\frac{3}{2}\right)\right]_{-3/2}^{1/2}$	1
		$=\frac{1}{4}\tan^{-1}\left(\frac{1}{2}+\frac{3}{2}\right)-\frac{1}{4}\tan^{-1}\left(-\frac{3}{2}+\frac{3}{2}\right)$	½
		$=\frac{1}{4}\tan^{-1}(2)$	½
		--	
	b)	Evaluate $\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt[3]{\cot x}}dx$	**04**
	Ans	$\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt[3]{\cot x}}dx$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	b)	$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt[3]{\cot x}} dx$	
		$= \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt[3]{\frac{\cos x}{\sin x}}} dx$	
		$= \int_{\pi/6}^{\pi/3} \frac{1}{1+\frac{\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}}} dx$	
		$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}} dx$ --------------------(1)	½
		$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt[3]{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt[3]{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}} dx$	1
		$\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt[3]{\cos\left(\frac{\pi}{2}-x\right)}} dx$	
		$\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}} dx$ --------------------(2)	½
		add (1) and (2)	
		$I + I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}} + \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}} dx$	
		$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}} dx$	½
		$2I = \int_{\pi/6}^{\pi/3} 1 dx$	½
		$2I = [x]_{\pi/6}^{\pi/3}$	
		$2I = \frac{\pi}{3} - \frac{\pi}{6}$	½
		$I = \frac{\pi}{12}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	c)	Find the area of the region enclosed between parabola $y = x^2 + 1$ and the line $y = 2x + 1$	**04**
	Ans	$y = x^2 + 1 \quad \text{and} \quad y = 2x + 1$	
		$\therefore x^2 + 1 = 2x + 1$	
		$x^2 - 2x = 0$	
		$x(x - 2) = 0$	
		$\therefore x = 0 \ \& \ x = 2$	1
		$\text{Area} = \int_a^b (y_1 - y_2)\,dx$	½
		$= \int_0^2 \left[(2x + 1) - (x^2 + 1)\right]dx$	
		$= \int_0^2 \left[2x + 1 - x^2 - 1\right]dx$	
		$= \int_0^2 \left[2x - x^2\right]dx$	
		$= \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$	1
		$= \left[2^2 - \frac{2^3}{3}\right] - [0]$	1
		$= \frac{4}{3}$	½
	d)	$\text{Solve } x^2.\frac{dy}{dx} = x^2 + xy + y^2$	**04**
	Ans	$x^2.\frac{dy}{dx} = x^2 + xy + y^2$	
		$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$	
		$\text{Put } y = vx$	
		$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$	½
		$v + x\frac{dv}{dx} = \frac{x^2 + x(vx) + (vx)^2}{x^2}$	½
		$v + x\frac{dv}{dx} = \frac{x^2 + vx^2 + v^2x^2}{x^2}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	d)	$v + x\frac{dv}{dx} = \frac{x^2(1+v+v^2)}{x^2}$	
		$v + x\frac{dv}{dx} = 1+v+v^2$	
		$x\frac{dv}{dx} = 1+v^2$	
		$xdv = (1+v^2)dx$	
		$\frac{1}{1+v^2}dv = \frac{1}{x}dx$	1
		$\int \frac{1}{1+v^2}dv = \int \frac{1}{x}dx$	½
		$\tan^{-1}(v) = \log x + c$	1
		$\tan^{-1}\left(\frac{y}{x}\right) = \log x + c$	½
		--	
	e)	Solve $\cos^2(x-2y) = 1 - 2\frac{dy}{dx}$	**04**
	Ans	$\cos^2(x-2y) = 1 - 2\frac{dy}{dx}$	
		Put $x - 2y = v$	
		$1 - 2\frac{dy}{dx} = \frac{dv}{dx}$	1
		$\therefore \cos^2 v = \frac{dv}{dx}$	
		$dx = \frac{1}{\cos^2 v}dv$	1
		$\therefore$ solution is	
		$\int dx = \int \sec^2 v dv$	½
		$x = \tan v + c$	1
		$x = \tan(x-2y) + c$	½
		--	
	f)	Solve $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$	**04**
	Ans	$\therefore \frac{dy}{dx} + \frac{1}{1+x^2}y = \frac{e^{\tan^{-1}x}}{1+x^2}$	
		Comparing with $\frac{dy}{dx} + Py = Q$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	f)	$\therefore P=\frac{1}{1+x^2}, Q=\frac{e^{\tan^{-1}x}}{1+x^2}$	½
		Integrating factor $= e^{\int P\,dx} = e^{\int\frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$	1
		$y.IF=\int Q.IF\,dx$	
		$ye^{\tan^{-1}x}=\int\frac{e^{\tan^{-1}x}}{1+x^2}e^{\tan^{-1}x}dx$	1
		$ye^{\tan^{-1}x}=\int\frac{\left(e^{\tan^{-1}x}\right)^2}{1+x^2}dx$	
		Put $\tan^{-1}x=t$ OR Put $e^{\tan^{-1}x}=t$	
		$\therefore \frac{1}{1+x^2}dx=dt$ $\therefore e^{\tan^{-1}x}.\frac{1}{1+x^2}dx=dt$	½
		$ye^{\tan^{-1}x}=\int\left(e^t\right)^2dt=\int e^{2t}dt$ $\therefore ye^{\tan^{-1}x}=\int t\,dt$	½
		$ye^{\tan^{-1}x}=\frac{e^{2t}}{2}+c$ $ye^{\tan^{-1}x}=\frac{t^2}{2}+c$	½
		$ye^{\tan^{-1}x}=\frac{e^{2\tan^{-1}x}}{2}+c$ $ye^{\tan^{-1}x}=\frac{\left(e^{\tan^{-1}x}\right)^2}{2}+c$	
		--	
4		**Attempt any FOUR of the following:**	**16**
	a)	Evaluate $\int_3^7\frac{(10-x)^2}{x^2+(10-x)^2}dx$	**04**
	Ans	Let $I=\int_3^7\frac{(10-x)^2}{x^2+(10-x)^2}dx$ --------------------(1)	
		$I=\int_3^7\frac{\left(10-(7+3-x)\right)^2}{(7+3-x)^2+\left(10-(7+3-x)\right)^2}dx$	1
		$I=\int_3^7\frac{(x)^2}{(10-x)^2+(x)^2}dx$ ----------------------(2)	
		Adding (1) and (2)	
		$I+I=\int_3^7\frac{(10-x)^2}{x^2+(10-x)^2}+\int_3^7\frac{x^2}{(10-x)^2+x^2}dx$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	a)	$2I=\int_3^7 \frac{(10-x)^2+x^2}{x^2+(10-x)^2}dx$	½
		$2I=\int_3^7 1dx$	½
		$2I=[x]_3^7$	½
		$2I=7-3$	
		$I=\frac{4}{2}$	
		$I=2$	½
	b)	Evaluate $\int_0^1 x.\sin^{-1}xdx$	**04**
	Ans	$\int_0^1 x.\sin^{-1}xdx$	
		$=\sin^{-1}x\int_0^1 x.dx-\int_0^1\left[\int_0^1 x.dx\frac{d\left(\sin^{-1}x\right)}{dx}\right]dx$	½
		$=\sin^{-1}x.\frac{x^2}{2}-\int_0^1\frac{x^2}{2}.\frac{1}{\sqrt{1-x^2}}dx$	½
		$=\frac{x^2.\sin^{-1}x}{2}+\frac{1}{2}\int_0^1\frac{1-x^2-1}{\sqrt{1-x^2}}dx$	½
		$=\frac{x^2.\sin^{-1}x}{2}+\frac{1}{2}\left[\int_0^1\left(\frac{1-x^2}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}\right)dx\right]$	
		$=\frac{x^2.\sin^{-1}x}{2}+\frac{1}{2}\left[\int_0^1\left(\sqrt{1-x^2}-\frac{1}{\sqrt{1-x^2}}\right)dx\right]$	
		$=\left[\frac{x^2.\sin^{-1}x}{2}+\frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1^2}{2}\sin^{-1}(x)-\sin^{-1}x\right]\right]_0^1$	1
		$=\left[\frac{x^2.\sin^{-1}x}{2}+\frac{x}{4}\sqrt{1-x^2}-\frac{1}{4}\sin^{-1}(x)\right]_0^1$	
		$=\left[\frac{1^2.\sin^{-1}(1)}{2}+0-\frac{1}{4}\sin^{-1}(1)\right]-0$	½
		$=\frac{\frac{\pi}{2}}{2}-\frac{1}{4}.\frac{\pi}{2}$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4	b)	$= \frac{\pi}{4} - \frac{\pi}{8}$	
		$= \frac{\pi}{8}$	1
		--	
	c)	Find the area of the region in the first quadrant enclosed by the X-axis the line $y = x$ and the circle $x^2 + y^2 = 8$	**04**
	Ans	$y = x \ , \ x^2 + y^2 = 8$	
		$\therefore x^2 + x^2 = 8$	
		$2x^2 = 8$	
		$x^2 = 4$	
		$\therefore x = \pm 2 \qquad \text{In first qudrant}$	
		$x = 0 \text{ to } x = 2$	1
		$A = \int_a^b (y_2 - y_1)\, dx$	½
		$\therefore A = \int_0^2 \left(\sqrt{\left(\sqrt{8}\right)^2 - x^2} - x \right) dx$	
		$\therefore A = \left[\frac{x}{2}\sqrt{\left(\sqrt{8}\right)^2 - x^2} + \frac{\left(\sqrt{8}\right)^2}{2} \sin^{-1}\left(\frac{x}{\sqrt{8}}\right) - \frac{x^2}{2} \right]_0^2$	1
		$\therefore A = \left[\frac{2}{2}\sqrt{\left(\sqrt{8}\right)^2 - (2)^2} + \frac{\left(\sqrt{8}\right)^2}{2} \sin^{-1}\left(\frac{2}{\sqrt{8}}\right) - \frac{(2)^2}{2} \right] - 0$	½
		$\therefore A = 2 + 4\left(\frac{\pi}{4}\right) - 2$	
		$\therefore A = \pi$	1
		--	
	d)	Solve $y^3.\sec^2 x dx + \left(3y^2 \tan x - \sec^2 y\right) dy = 0$	**04**
	Ans	Comparing with $\int M dx + \int N dy = c$	
		$M = y^3.\sec^2 x \qquad N = 3y^2 \tan x - \sec^2 y$	
		$\frac{\partial M}{\partial y} = 3y^2 \sec^2 x \qquad \frac{\partial N}{\partial x} = 3y^2 \sec^2 x$	1
		$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	d)	D.E. is exact	
		$\therefore$ Solution is	
		$\int y^3.\sec^2 x dx + \int -\sec^2 y dy = c$	1
		$y^3 \tan x - \tan y = c$	1
		--	
	e)	Solve $(x+y+1)^2 \frac{dy}{dx} = 1$	**04**
	Ans	Put $x+y+1 = v$	
		$\therefore 1 + \frac{dy}{dx} = \frac{dv}{dx}$	1
		$\therefore \frac{dy}{dx} = \frac{dv}{dx} - 1$	1
		$\therefore v^2\left(\frac{dv}{dx} - 1\right) = 1$	
		$\therefore v^2 \frac{dv}{dx} - v^2 = 1$	
		$\therefore v^2 \frac{dv}{dx} = 1 + v^2$	
		$\therefore \left(\frac{v^2}{1+v^2}\right) dv = dx$	
		$\therefore$ solution is	
		$\int \left(\frac{v^2}{1+v^2}\right) dv = \int dx$	½
		$\int \left(\frac{1+v^2-1}{1+v^2}\right) dv = \int dx$	
		$\int \left(1 - \frac{1}{1+v^2}\right) dv = \int dx$	
		$\therefore v - \tan^{-1} v = x + c$	1
		$\therefore x + y + 1 - \tan^{-1}(x+y+1) = x + c$	½
		$\therefore y + 1 - \tan^{-1}(x+y+1) = c$	
		--	
	f)	Verify that $y = e^{m\sin^{-1}x}$ is a solution of differential equation	**04**
		$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2 y = 0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4	Ans	Consider $y = e^{m\sin^{-1}x}$	
		$\frac{dy}{dx} = e^{m\sin^{-1}x}.m\frac{1}{\sqrt{1-x^2}}$	½
		$\frac{dy}{dx} = \frac{my}{\sqrt{1-x^2}}$	
		$\sqrt{1-x^2}\frac{dy}{dx} = my$	
		Squaring,	
		$\therefore \left(1-x^2\right)\left(\frac{dy}{dx}\right)^2 = m^2y^2$	½
		$\therefore \left(1-x^2\right)2\frac{dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\left(-2x\right) = m^2\left(2y\frac{dy}{dx}\right)$	1
		$\therefore 2\frac{dy}{dx}\left(\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx}\right) = 2m^2y\frac{dy}{dx}$	1
		$\therefore \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$	
		$\therefore \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$	1
		OR	
		$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$	
		Consider $y = e^{m\sin^{-1}x}$	
		$\therefore \frac{dy}{dx} = e^{m\sin^{-1}x}.m\frac{1}{\sqrt{1-x^2}}$	½
		$\therefore \frac{dy}{dx} = \frac{my}{\sqrt{1-x^2}}$	
		$\therefore \sqrt{1-x^2}\frac{dy}{dx} = my$	
		$\therefore \sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{1}{2\sqrt{1-x^2}}\left(-2x\right) = m\frac{dy}{dx}$	1
		$\therefore \left(\sqrt{1-x^2}\right)^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m\sqrt{1-x^2}\frac{dy}{dx}$	1
		$\therefore \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m\left(my\right)$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4		$\therefore \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=m^2y$	
		$\therefore \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$	1
5		**Attempt any <u>FOUR</u> of the following:**	**16**
	a)	A Card is drawn at random from a well shuffled pack of 52 playing cards. A=event that the card drawn is not a spade. B=event that the card drawn is king. Verify that events A and B are independent.	**04**
	Ans	$n(s)={}^{52}C_1=52$	
		$n(A)={}^{39}C_1=39$	½
		$n(B)={}^{4}C_1=4$	½
		$p(A)=\frac{n(A)}{n(s)}=\frac{39}{52}=\frac{3}{4}$	½
		$p(B)=\frac{n(B)}{n(s)}=\frac{4}{52}=\frac{1}{13}$	
		Consider $p(A)\times p(B)=\frac{3}{4}\times\frac{1}{13}=\frac{3}{52}$	½
		$A\cap B$ = Event that the card drawn is a king of heart orof diamond or of club	
		$n(A\cap B)={}^{3}C_1=3$	
		$p(A\cap B)=\frac{n(A\cap B)}{n(s)}=\frac{3}{52}$	1
		$\therefore p(A\cap B)=p(A)\times p(B)$	½
		$\therefore$ A and B Independent events.	½
	b)	Assuming that the probability of a fatal accident during the year is $\frac{1}{1200}$. Calculate the probability that in a factory employing 300 workers there will be at least two fatal accidents in a year. $\left[e^{-0.25}=0.7788\right]$	**04**
	Ans	Given $n=300,\ p=\frac{1}{1200}$	
		$m=np=\frac{300}{1200}=0.25$	1
		$p(\text{at least two fatal accidents})=1-\left[p(0)+p(1)\right]$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	b)	$p\left(\text{at least two fatal accidents}\right)=1-\left[\frac{e^{-0.25}(.025)^0}{0!}+\frac{e^{-0.25}(.025)^1}{1!}\right]$	1
		$=1-\left[0.7788+0.7788\times 0.25\right]$ $=0.0265$	1
		--	
	c)	In certain examination 500 students appererd.Mean score is 68 with S.D 8.Find the number of students scoring i) less than 50 ii) more than 60	04
	Ans	Given $\overline{x}=68$, $\sigma=8$	
		i) when $x=50$ $z=\frac{x-\overline{x}}{\sigma}=\frac{50-68}{8}$ $=-2.25$	2
		ii) when $x=60$ $z=\frac{x-\overline{x}}{\sigma}=\frac{60-68}{8}$ $=-1$	2
		<u>NOTE</u> : As the areas for the above problem are not given, the students cannot solve the problem completely. If students attempted to solve the problem and calculated upto value z. Full marks to be rewarded.	
		--	
	d)	Evaluate $\int\frac{1}{4+5\sin\left(2x\right)}dx$	04
	Ans	$\int\frac{1}{4+5\sin\left(2x\right)}dx$ Put $\tan x=t$, $dx=\frac{dt}{1+t^2}$, $\sin 2x=\frac{2t}{1+t^2}$	
		$=\int\frac{\frac{dt}{1+t^2}}{4+5\left(\frac{2t}{1+t^2}\right)}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	d)	$=\int \frac{dt}{4+4t^2+10t}$	
		$=\int \frac{dt}{4t^2+10t+4}$	
		$=\int \frac{dt}{4t^2+10t+\frac{100}{16}-\frac{100}{16}+4}$	½
		$=\int \frac{dt}{\left(2t+\frac{5}{2}\right)^2-\left(\frac{3}{2}\right)^2}$	½
		$=\frac{1}{2\times\frac{3}{2}}\log\left\|\frac{2t+\frac{5}{2}-\frac{3}{2}}{2t+\frac{5}{2}+\frac{3}{2}}\right\|\times\frac{1}{2}+c$	1
		$=\frac{1}{6}\log\left\|\frac{2t+1}{2t+4}\right\|+c$	½
		$=\frac{1}{6}\log\left\|\frac{2\tan x+1}{2\tan x+4}\right\|+c$	½
		OR	
		$\int \frac{1}{4+5\sin(2x)}dx$	
		Put $\tan x=t$, $dx=\frac{dt}{1+t^2}$, $\sin 2x=\frac{2t}{1+t^2}$	
		$=\int \frac{\frac{dt}{1+t^2}}{4+5\left(\frac{2t}{1+t^2}\right)}$	1
		$=\int \frac{dt}{4+4t^2+10t}$	
		$\int \frac{dt}{4t^2+10t+4}$	
		$=\frac{1}{4}\int \frac{dt}{t^2+\frac{5}{2}t+1}$	
		$=\frac{1}{4}\int \frac{dt}{t^2+\frac{5}{2}t+\frac{25}{16}+1-\frac{25}{16}}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	d)	$= \frac{1}{4}\int \frac{dt}{\left(t+\frac{5}{4}\right)^2-\left(\frac{3}{4}\right)^2}$	½
		$= \frac{1}{4}\times\frac{1}{2\times\frac{3}{4}}\log\left\lvert\frac{t+\frac{5}{4}-\frac{3}{4}}{t+\frac{5}{4}+\frac{3}{4}}\right\rvert$	1
		$= \frac{1}{6}\log\left\lvert\frac{2t+1}{2t+4}\right\rvert$	½
		$= \frac{1}{6}\log\left\lvert\frac{2\tan x+1}{2\tan x+4}\right\rvert$	½
		---	
	e)	Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin(2x)}{4-\sin^2 x}dx$	**04**
	Ans	$\int_0^{\frac{\pi}{2}} \frac{\sin(2x)}{4-\sin^2 x}dx$	
		$= \int_0^{\frac{\pi}{2}} \frac{2\sin x\cos x}{4-\sin^2 x}dx$	
		Put $\sin x = t$ when $x = 0$ $t = 0$	
		$\cos x dx = dt$ when $x = \frac{\pi}{2}$ $t = 1$	1
		$= \int_0^1 \frac{2tdt}{4-t^2}$	
		consider, $\frac{2t}{4-t^2} = \frac{2t}{(2+t)(2-t)}$	
		$\frac{2t}{(2+t)(2-t)} = \frac{A}{2+t}+\frac{B}{2-t}$	
		$\therefore\ 2t = A(2-t)+B(2+t)$	
		put $t = -2$	
		$\therefore\ A = -1$	½
		put $t = 2$	
		$B = 1$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	e)	$\therefore \int_0^1 \frac{2tdt}{4-t^2} = \int_0^1 \left(\frac{-1}{2+t} + \frac{1}{2-t} \right) dt$	1
		$= \left[-\log(2+t) - \log(2-t) \right]_0^1$	
		$= (-\log 3 - \log 1) - (-\log 2 - \log 2)$	1
		$= -\log 3 + \log 4$	
		OR	
		$\int_0^{\frac{\pi}{2}} \frac{\sin(2x)}{4-\sin^2 x} dx$	
		$= \int_0^{\frac{\pi}{2}} \frac{2\sin x \cos x}{4-\sin^2 x} dx$	
		Put $\sin x = t$ when $x = 0$ $t = 0$	
		$\cos x dx = dt$ when $x = \frac{\pi}{2}$ $t = 1$	1
		$= \int_0^1 \frac{2tdt}{4-t^2}$	
		$= -\int_0^1 \frac{-2tdt}{4-t^2}$	
		$= -\left[\log(4-t^2) \right]_0^1$	1
		$= -\left[\log(4-1^2) - \log(4-0^2) \right]$	1
		$= -\log(3) + \log(4)$	1
		--	
	f)	Solve $\left(2x + e^x \log y\right) dx + \left(\frac{e^x}{y} + 1 \right) dy = 0$	**04**
	Ans	comparing with $Mdx + Ndy = 0$	
		$\therefore M = 2x + e^x \log y$ $\quad N = \frac{e^x}{y} + 1$	
		$\therefore \frac{\partial M}{\partial y} = \frac{e^x}{y}$, $\frac{\partial N}{\partial x} = \frac{e^x}{y}$	½ +½
		$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	1
		$\therefore$ given D.E.equation is exact	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5		$\therefore$ solution is	
		$\int_{y-constant} M\,dx + \int_{terms\ free\ from\ x} N\,dy = c$	
		$\int_{y-constant} \left(2x + e^{x}\log y\right)dx + \int 1dy = c$	1
		$\therefore \frac{2x^{2}}{2} + e^{x}\log y + y = c$	
		$\therefore x^{2} + e^{x}\log y + y = c$	1
6		**Attempt any FOUR of the following:**	**16**
	a)	A Husband and wife appeared in an interview for two vacancies in an office The Probability of husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$ Find the probability that i) both of them are selected. ii) only one of them is selected.	**04**
	Ans	Given $p(H) = \frac{1}{7}$ $p(W) = \frac{1}{5}$	
		$p\left(H'\right) = 1 - p(H) = \frac{6}{7}$ and $p\left(W'\right) = 1 - p(W) = \frac{4}{5}$	1
		$i)\ p(H \cap W) = p(H)\,p(W) = \frac{1}{7}\times\frac{1}{5} = \frac{1}{35}$	1
		$ii)\ p\left(H \cap W'\right) + p\left(H' \cap W\right) = p(H)\times p\left(W'\right) + p\left(H'\right)\times p(W)$	
		$= \frac{1}{7}\times\frac{4}{5} + \frac{6}{7}\times\frac{1}{5}$	1
		$= \frac{2}{7}$	1
	b)	A company manufactures electric motors. The probability that an electric motor is defective is 0.01. What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors? $\left(e^{-3} = 0.0498\right)$	**04**
	Ans	Given $n = 300,\ p = 0.01$	
		$m = np = 300 \times 0.01 = 3$	1
		$P(5) = \frac{e^{-3}3^{5}}{5!}$	2
		$= 0.1008$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	c)	In a test of 2000 electric bulbs it was found that the life of particular make was normally distributed with average life of 2040 hrs. and S.D.of 60 hrs. Estimate the number of bulbs likely to burn for (i) between 1920hrs. and 2160 hrs. (ii) more than 2150 hrs.	**04**
	Ans	Given that $A(2) = 0.4772$ $A(1.83) = 0.4664$ $\bar{x} = 2040, \sigma = 60$ i) $x = 1920, \quad x = 2160$ $z = \frac{x - \bar{x}}{\sigma} = \frac{1920 - 2040}{60} = -2$ $z = \frac{x - \bar{x}}{\sigma} = \frac{2160 - 2040}{60} = 2$	½
		$P(\text{Between 1920 hrs. and 2160 hrs.}) = A(-2) + A(2) = 0.4772 + 0.4772$ $= 0.9544$	1
		Number of bulbs having life between 1920 hrs. and 2160 hrs. $= 0.9544 \times 2000$ $= 1908.8 \approx 1909$	½
		ii) $x = 2150$ $z = \frac{x - \bar{x}}{S.D} = \frac{2150 - 2040}{60} = 1.83$	½
		$P(\text{More than 2150 hrs.}) = 0.5 - A(1.83)$ $= 0.5 - 0.4664$ $= 0.0336$	1
		Number of bulbs having life more than 2150 hrs. $= 0.0336 \times 2000 = 67.2 \approx 67$	½
	d)	Find the maximum and minimum values of $2x^3 - 3x^2 - 36x + 10$	
	Ans	$y = 2x^3 - 3x^2 - 36x + 10$ $\frac{dy}{dx} = 6x^2 - 6x - 36$	½
		$\frac{d^2y}{dx^2} = 12x - 6$	½
		$\frac{dy}{dx} = 0$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6	d)	$6x^2-6x-36=0$ or $x^2-x-6=0$ $x=3$ or $x=-2$ $for\ x=3$ $\frac{d^2y}{dx^2}=12(3)-6=30$ $\therefore$ y is minimum at $x=3$ $y_{\min}=2(3)^3-3(3)^2-36(3)+10=-71$ $for\ x=-2$ $\frac{d^2y}{dx^2}=12(-2)-6=-30$ $\therefore$ y is maximum at $x=-2$ $y_{\max}=2(-2)^3-3(-2)^2-36(-2)+10=54$	1 ½ ½ ½ ½
	e) Ans	Find the equation of the tangent and the normal to the curve $2x^2-xy+3y^2=18$ at $(3,1)$ $2x^2-xy+3y^2=18$ at $(3,1)$ $\therefore\ 4x-\left[x\frac{dy}{dx}+y\right]+6y\frac{dy}{dx}=0$ $4x-x\frac{dy}{dx}-y+6y\frac{dy}{dx}=0$ $\frac{dy}{dx}=\frac{y-4x}{6y-x}$ at $(3,1)$ $\frac{dy}{dx}=\frac{1-4(3)}{6(1)-3}=-\frac{11}{3}$ $\therefore$ slope of tangent $=-\frac{11}{3}$ Equation of tangent is $y-1=-\frac{11}{3}(x-3)$ $3y-3=-11x+33$ $11x+3y-36=0$ $\therefore$ slope of normal $=\frac{-1}{\frac{dy}{dx}}=\frac{3}{11}$ Equation of normal is	**04** 1 ½ 1 ½

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	e)	$y-1=\frac{3}{11}(x-3)$	
		$11y-11=3x-9$	
		$3x-11y+2=0$	1
		--	
	f)	Find by integration the area of the ellipse $4x^2+9y^2=36$	**04**
	Ans	$4x^2+9y^2=36$	
		$\frac{x^2}{9}+\frac{y^2}{4}=1$	
		$\therefore\ y^2=\frac{4}{9}\left(9-x^2\right)$	
		$\therefore\ y=\frac{2}{3}\sqrt{3^2-x^2}$	½
		$Area=4\int_a^b ydx$	
		$=4\int_0^3\frac{2}{3}\sqrt{3^2-x^2}dx$	1
		$=\frac{8}{3}\int_0^3\sqrt{3^2-x^2}dx$	
		$=\frac{8}{3}\left[\frac{x}{2}\sqrt{(3)^2-(x)^2}+\frac{(3)^2}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$	1
		$=\frac{8}{3}\left[\frac{3}{2}\sqrt{(3)^2-(3)^2}+\frac{(3)^2}{2}\sin^{-1}\left(\frac{3}{3}\right)\right]-[0]$	½
		$=\frac{8}{3}\left[0+\frac{9}{2}\sin^{-1}(1)\right]$	
		$=\frac{8}{3}\times\frac{9}{2}\times\frac{\pi}{2}$	
		$=6\pi$	1
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.*	
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