SUMMER– 17 EXAMINATION

Model Answer

Subject Code: **17216**

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1		**Solve any TEN of the following:**	**20**
	a)	Find the value of : $i^{20}+i^{30}+i^{40}+i^{50}$	**02**
	Ans	$i^{20}+i^{30}+i^{40}+i^{50}$	½
		$=\left(i^2\right)^{10}+\left(i^2\right)^{15}+\left(i^2\right)^{20}+\left(i^2\right)^{25}$	½
		$=\left(-1\right)^{10}+\left(-1\right)^{15}+\left(-1\right)^{20}+\left(-1\right)^{25}$	½
		$=1-1+1-1$	½
		$=0$	
		--	
	b)	Express : $(2+3i)(1-4i)$ in the form $a+ib$	**02**
	Ans	$(2+3i)(1-4i)$	
		$=2-8i+3i-12i^2$	1
		$=2-5i-12(-1)$	
		$=2-5i+12$	1
		$=14-5i$	
		--	
	c)	Find 'a' if $f(x)=ax+10$ and $f(1)=13$	**02**
	Ans	$f(x)=ax+10$	
		$\therefore f(1)=a(1)+10$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	c)	$\therefore 13 = a + 10$ $\therefore 13 - 10 = a$ $\therefore a = 3$	½ 1
	d) Ans	Define : Even and odd function Even function:- If $f(-x) = f(x)$, then the function is an even function Odd function:- If $f(-x) = -f(x)$, then the function is an odd function	**02** 1 1
	e) Ans	Evaluate : $\lim_{x \to 3} \frac{\sqrt{x} + \sqrt{3}}{x + 3}$ $\lim_{x \to 3} \frac{\sqrt{x} + \sqrt{3}}{x + 3}$ $= \frac{\sqrt{3} + \sqrt{3}}{3 + 3}$ $= \frac{2\sqrt{3}}{6}$ $= \frac{1}{\sqrt{3}}$	**02** 1 1
	f) Ans	Evaluate : $\lim_{x \to 0} x.\operatorname{cosec} x$ $\lim_{x \to 0} x.\operatorname{cosec} x$ $= \lim_{x \to 0} \frac{x}{\sin x}$ $= 1$	**02** 1 1
	g) Ans	Evaluate : $\lim_{x \to 0} \frac{a^x + b^x - 2}{x}$ $\lim_{x \to 0} \frac{a^x + b^x - 2}{x}$ $= \lim_{x \to 0} \frac{a^x - 1 + b^x - 1}{x}$	**02** 1

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	g)	$= \lim_{x\to 0}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}\right)$ $= \lim_{x\to 0}\frac{a^x-1}{x}+\lim_{x\to 0}\frac{b^x-1}{x}$ $= \log a + \log b$ $= \log ab$	 1
	h) Ans	Evaluate: $\lim_{x\to 0}\frac{\log(1+5x)}{x}$ $\lim_{x\to 0}\frac{\log(1+5x)}{x}$ $= \lim_{x\to 0}\frac{1}{x}\log(1+5x)$ $= \lim_{x\to 0}\log(1+5x)^{\frac{1}{x}}$ $= \log\left[\lim_{x\to 0}(1+5x)^{\frac{1}{x}}\right]$ $= \log\left[\lim_{x\to 0}(1+5x)^{\frac{1}{5x}}\right]^5$ $= \log e^5$ $= 5\log e$ $= 5$	**02** ½ ½ 1
	i) Ans	If $y = 2e^{3x}+\tan x-\cos 2x+9\sin^{-1}x$, find $\frac{dy}{dx}$ $y = 2e^{3x}+\tan x-\cos 2x+9\sin^{-1}x$ $\frac{dy}{dx} = 2(3)e^{3x}+\sec^2 x-(-\sin 2x.2)+9.\frac{1}{\sqrt{1-x^2}}$ $\therefore \frac{dy}{dx} = 6e^{3x}+\sec^2 x+2\sin 2x+\frac{9}{\sqrt{1-x^2}}$	**02** 1 1
	j) Ans	If $y = \frac{\log x}{x}$, find $\frac{dy}{dx}$ $y = \frac{\log x}{x}$	**02**

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1	j)	$\frac{dy}{dx}=\frac{x\frac{d}{dx}(\log x)-\log x\frac{d}{dx}(x)}{x^2}$	½
		$\therefore\frac{dy}{dx}=\frac{x\frac{1}{x}-\log x.1}{x^2}$	1
		$\therefore\frac{dy}{dx}=\frac{1-\log x}{x^2}$	½
		--	
	k)	Differentiate $7x^5-11x^2$ w.r.t. $7x^2-15$	**02**
	Ans	Let $u=7x^5-11x^2$, $v=7x^2-15x$	
		$\therefore\frac{du}{dx}=35x^4-22x$ and $\frac{dv}{dx}=14x-15$	½ +½
		$\because\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{35x^4-22x}{14x-15}$	1
		--	
	l)	Differentiate w.r.t. : $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$	**02**
	Ans	Let $y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$	
		Put $x=\tan\theta$ $\therefore\theta=\tan^{-1}x$	
		$\therefore y=\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$	½
		$\therefore y=\tan^{-1}(\tan 2\theta)$	½
		$\therefore y=2\theta$	
		$\therefore y=2\tan^{-1}x$	½
		$\therefore\frac{dy}{dx}=2\left(\frac{1}{1+x^2}\right)$	½
		$\therefore\frac{dy}{dx}=\frac{2}{1+x^2}$	
		OR	
		$y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$	
		$\therefore\tan y=\frac{2x}{1-x^2}$	½

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1	l)	$\therefore \sec^2 y \frac{dy}{dx} = \frac{(1-x^2)2 - 2x(-2x)}{(1-x^2)^2}$	1
		$\therefore \sec^2 y \frac{dy}{dx} = \frac{2-2x^2+4x^2}{(1-x^2)^2}$	
		$\therefore \sec^2 y \frac{dy}{dx} = \frac{2+2x^2}{(1-x^2)^2}$	
		$\therefore \frac{dy}{dx} = \frac{2(1+x^2)}{\sec^2 y (1-x^2)^2}$	½
	m)	Prove that the root of the equation $x^3 - x - 4 = 0$ lies between 0 and 2	**02**
	Ans	Let $f(x) = x^3 - x - 4$	
		$f(0) = -4 < 0$	1
		$f(2) = 2 > 0$	1
		$\therefore$ root lies between 0 and 2	
	n)	Find the first iteration by using Jacobi's method for the following system of equations: $10x + y + 2z = 13$, $3x + 10y + z = 14$, $2x + 3y + 10z = 15$	**02**
	Ans	$x = \frac{13 - y - 2z}{10}$	
		$y = \frac{14 - 3x - z}{10}$	½
		$z = \frac{15 - 2x - 3y}{10}$	
		Let $x_0 = y_0 = z_0 = 0$	½
		$\therefore x_1 = 1.3$	½
		$y_1 = 1.4$	½
		$z_1 = 1.5$	
2		**Solve any FOUR of the following:**	**16**
	a)	If $f(x) = ax^2 + bx + 2$ and $f(1) = 3$, $f(4) = 42$, find a and b	**04**
	Ans	$f(x) = ax^2 + bx + 2$	
		$\therefore f(1) = a(1)^2 + b(1) + 2$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
2	a)	$\therefore 3 = a + b + 2$	
		$\therefore a + b = 1$	1
		$\therefore f(4) = a(4)^2 + b(4) + 2$	
		$\therefore 42 = 16a + 4b + 2$	
		$\therefore 40 = 16a + 4b$	
		$\therefore 4a + b = 10$	1
		$\therefore a + b = 1$	
		$4a + b = 10$	
		$-\quad -\quad -$	

		$-3a = -9$	1
		$\therefore a = 3$	
		$\therefore b = -2$	1
		--	
	b)	If $f(x) = \dfrac{2x+3}{3x-2}$ prove that $f\left[f(x)\right] = x$	**04**
	Ans	$f\left[f(x)\right] = f\left(\dfrac{2x+3}{3x-2}\right)$	½
		$= \dfrac{2\left(\dfrac{2x+3}{3x-2}\right)+3}{3\left(\dfrac{2x+3}{3x-2}\right)-2}$	1
		$= \dfrac{\dfrac{2(2x+3)+3(3x-2)}{3x-2}}{\dfrac{3(2x+3)-2(3x-2)}{3x-2}}$	1
		$= \dfrac{4x+6+9x-6}{6x+9-6x+4}$	
		$= \dfrac{13x}{13}$	1
		$= x$	½
		$\therefore f\left[f(x)\right] = x$	
		--	
	c)	Separate into real and imaginary parts of : $\dfrac{2+i}{(3-i)(1+2i)}$	**04**
	Ans	$\dfrac{2+i}{(3-i)(1+2i)}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	c)	$=\frac{2+i}{3+6i-i-2i^2}$ $=\frac{2+i}{3+5i+2}$ $=\frac{2+i}{5+5i}$ $=\frac{2+i}{5+5i}\times\frac{5-5i}{5-5i}$ $=\frac{10-10i+5i-5i^2}{(5)^2-(5i)^2}$ $=\frac{10-5i-5(-1)}{25+25}$ $=\frac{10-5i+5}{50}$ $=\frac{15-5i}{50}$ $=\frac{3-i}{10}$ $=\frac{3}{10}-\frac{i}{10}$ $\therefore$ Real part $=\frac{3}{10}$ Imaginary part $=\frac{-1}{10}$	1 ½ ½ 1 ½ ½
	d) Ans	Solve : $(4-5i)x+(2+3i)y=10-7i$ $(4-5i)x+(2+3i)y=10-7i$ $\therefore 4x-5ix+2y+3iy=10-7i$ $(4x+2y)+(-5x+3y)i=10-7i$ $\therefore 4x+2y=10$ $-5x+3y=-7$ $\therefore 20x+10y=50$ $-20x+12y=-28$ ----------- $22y=22$ $y=1$ $x=2$	**04** 1 1 1 1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
2	e)	Simplify: $\dfrac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}}$	**04**
	Ans	$\dfrac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}}$	
		$= \dfrac{(\cos\theta + i\sin\theta)^{12} (\cos\theta + i\sin\theta)^{-20}}{(\cos\theta + i\sin\theta)^{12} (\cos\theta + i\sin\theta)^{-20}}$	2 2
		$= 1$	
		--	
	f)	Find all the cube roots of (-1)	**04**
	Ans	Let $x = \sqrt[3]{-1}$	
		$\therefore x^3 = -1$	
		Let $z = -1$	
		$\therefore z = -1 + 0i$	
		$\mathrm{Re}(z) = -1, \mathrm{Im}(z) = 0$	
		$r = \lvert z \rvert = \sqrt{(-1)^2 + 0} = 1$	½
		$\theta = \pi - \tan^{-1}\left(\left\lvert \frac{0}{1} \right\rvert\right)$	
		$\theta = \pi - 0 = \pi$	½
		$z = r(\cos\theta + i\sin\theta)$	
		$z = 1(\cos\pi + i\sin\pi)$	½
		In general polar form , $z = r(\cos(2\pi k + \theta) + i\sin(2\pi k + \theta))$	
		$z = 1(\cos(2\pi k + \pi) + i\sin(2\pi k + \pi))$	½
		$z^{\frac{1}{3}} = (\cos(2\pi k + \pi) + i\sin(2\pi k + \pi))^{\frac{1}{3}}$	
		$z^{\frac{1}{3}} = \cos\left(\frac{2\pi k + \pi}{3}\right) + i\sin\left(\frac{2\pi k + \pi}{3}\right)$; k = 0,1,2	½
		when k = 0	
		$z_1 = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$	½
		when k = 1	
		$z_2 = \cos\pi + i\sin\pi = -1 + 0 = -1$	½
		when k = 2	
		$z_3 = \cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)$	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
3		**Solve any FOUR of the following:**	**16**
	a)	If $f(x) = \log[1+\tan x]$, show that $f\left(\frac{\pi}{4}-x\right) = \log 2 - f(x)$	**04**
	Ans	$f\left(\frac{\pi}{4}-x\right) = \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]$	½
		$= \log\left[1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]$	1
		$= \log\left[1+\frac{1-\tan x}{1+\tan x}\right]$	½
		$= \log\left[\frac{1+\tan x+1-\tan x}{1+\tan x}\right]$	
		$= \log\left[\frac{2}{1+\tan x}\right]$	1
		$= \log 2 - \log[1+\tan x]$	½
		$= \log 2 - f(x)$	½
		---	
	b)	If $f(x) = x^2 - 3x + 4$, then solve $f(1-x) = f(2x+1)$	**04**
	Ans	$f(1-x) = (1-x)^2 - 3(1-x) + 4$	
		$= 1 - 2x + x^2 - 3 + 3x + 4$	
		$= x^2 + x + 2$	1
		$f(2x+1)$	
		$= (2x+1)^2 - 3(2x+1) + 4$	
		$= 4x^2 + 4x + 1 - 6x - 3 + 4$	
		$= 4x^2 - 2x + 2$	1
		Given $f(1-x) = f(2x+1)$	
		$\therefore x^2 + x + 2 = 4x^2 - 2x + 2$	½
		$\therefore -3x^2 + 3x = 0$	
		$\therefore 3x^2 - 3x = 0$	
		$3x(x-1) = 0$	½
		$\therefore x = 0, 1$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	c)	Evaluate: $\lim_{x\to 5} \frac{x^2-9x+20}{x^2-6x+5}$	**04**
	Ans	$\lim_{x\to 5} \frac{x^2-9x+20}{x^2-6x+5}$	
		$=\lim_{x\to 5} \frac{(x-4)(x-5)}{(x-1)(x-5)}$	2
		$=\lim_{x\to 5} \frac{(x-4)}{(x-1)}$	
		$=\frac{5-4}{5-1}$	1
		$=\frac{1}{4}$	1
		---	
	d)	Evaluate: $\lim_{x\to 3} \frac{\sqrt{x^2+1}-\sqrt{10}}{x-3}$	**04**
	Ans	$\lim_{x\to 3} \frac{\sqrt{x^2+1}-\sqrt{10}}{x-3}$	
		$=\lim_{x\to 3} \frac{\sqrt{x^2+1}-\sqrt{10}}{x-3}\times\frac{\sqrt{x^2+1}+\sqrt{10}}{\sqrt{x^2+1}+\sqrt{10}}$	1
		$=\lim_{x\to 3} \frac{x^2+1-10}{(x-3)\left(\sqrt{x^2+1}+\sqrt{10}\right)}$	½
		$=\lim_{x\to 3} \frac{x^2-9}{(x-3)\left(\sqrt{x^2+1}+\sqrt{10}\right)}$	
		$=\lim_{x\to 3} \frac{(x-3)(x+3)}{(x-3)\left(\sqrt{x^2+1}+\sqrt{10}\right)}$	1
		$=\lim_{x\to 3} \frac{x+3}{\sqrt{x^2+1}+\sqrt{10}}$	
		$=\frac{3+3}{\sqrt{(3)^2+1}+\sqrt{10}}$	½
		$=\frac{6}{2\sqrt{10}}$	
		$=\frac{3}{\sqrt{10}}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	e)	Evaluate: $\lim_{x\to a}\frac{\sin x-\sin a}{x-a}$	04
	Ans	$\lim_{x\to a}\frac{\sin x-\sin a}{x-a}$ Put $x=a+h$ as $x\to a$, $h\to 0$ $=\lim_{h\to 0}\frac{\sin(a+h)-\sin a}{a+h-a}$	1
		$=\lim_{h\to 0}\frac{2\cos\left(\frac{a+h+a}{2}\right)\sin\left(\frac{a+h-a}{2}\right)}{h}$	1
		$=2\lim_{h\to 0}\frac{\cos\left(\frac{2a+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$ $=2\left(\lim_{h\to 0}\cos\left(\frac{2a+h}{2}\right)\right)\left(\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}.\frac{1}{2}\right)$	1
		$=2(\cos a)\frac{1}{2}$ $=\cos a$	1
	f)	Evaluate: $\lim_{x\to 0}\frac{15^x-5^x-3^x+1}{x.\sin x}$	04
	Ans	$\lim_{x\to 0}\frac{15^x-5^x-3^x+1}{x.\sin x}$ $=\lim_{x\to 0}\frac{5^x3^x-5^x-3^x+1}{x.\sin x}$ $=\lim_{x\to 0}\frac{5^x(3^x-1)-(3^x-1)}{x.\sin x}$ $=\lim_{x\to 0}\frac{(5^x-1)(3^x-1)}{x.\sin x}$	1
		$=\lim_{x\to 0}\frac{\frac{(5^x-1)(3^x-1)}{x^2}}{\frac{x.\sin x}{x^2}}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3	f)	$= \dfrac{\left(\lim\limits_{x\to 0}\dfrac{5^x-1}{x}\right)\left(\lim\limits_{x\to 0}\dfrac{3^x-1}{x}\right)}{\lim\limits_{x\to 0}\dfrac{\sin x}{x}}$	1
		$= (\log 5)(\log 3)$	1
		--	
4		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	Differentiate w.r.t. x : $x^{\sin 2x}$	**04**
	Ans	Consider $y = x^{\sin 2x}$	½
		$\therefore \log y = \log x^{\sin 2x}$	½
		$\therefore \log y = \sin 2x \log x$	
		$\therefore \dfrac{1}{y}\dfrac{dy}{dx} = \sin 2x \dfrac{1}{x} + \log x(\cos 2x)(2)$	2
		$\therefore \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\sin 2x}{x} + 2\log x(\cos 2x)$	
		$\therefore \dfrac{dy}{dx} = y\left[\dfrac{\sin 2x}{x} + 2\log x(\cos 2x)\right]$	1
		$\therefore \dfrac{dy}{dx} = x^{\sin 2x}\left[\dfrac{\sin 2x}{x} + 2\log x(\cos 2x)\right]$	
		--	
	b)	If $x = 3\cos\theta - \cos 3\theta$, $y = 3\sin\theta - \sin 3\theta$, then find $\dfrac{dy}{dx}$	**04**
	Ans	$x = 3\cos\theta - \cos 3\theta$	
		$\dfrac{dx}{d\theta} = -3\sin\theta + 3\sin 3\theta$	1½
		$y = 3\sin\theta - \sin 3\theta$	
		$\therefore \dfrac{dy}{d\theta} = 3\cos\theta - 3\cos 3\theta$	1½
		$\therefore \dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{3\cos\theta - 3\cos 3\theta}{-3\sin\theta + 3\sin 3\theta}$	1
		$\therefore \dfrac{dy}{dx} = \dfrac{\cos\theta - \cos 3\theta}{-\sin\theta + \sin 3\theta}$	
		--	

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4	c) Ans	Differentiate w.r.t. x $(\tan x)^x$ Consider $y = (\tan x)^x$ $\therefore \log y = \log(\tan x)^x$ $\therefore \log y = x\log(\tan x)$ $\therefore \frac{1}{y}\frac{dy}{dx} = x\frac{1}{\tan x}\sec^2 x + \log(\tan x)(1)$ $\therefore \frac{1}{y}\frac{dy}{dx} = \frac{x\sec^2 x}{\tan x} + \log(\tan x)$ $\therefore \frac{dy}{dx} = y\left[\frac{x\sec^2 x}{\tan x} + \log(\tan x)\right]$ $\therefore \frac{dy}{dx} = (\tan x)^x\left[\frac{x\sec^2 x}{\tan x} + \log(\tan x)\right]$	04 ½ ½ 2 1
		---	
	d) Ans	Differentiate $x^{\sin^{-1}x}$ w.r.t. $\sin^{-1}x$ Let $u = x^{\sin^{-1}x}$, $v = \sin^{-1}x$ Consider $u = x^{\sin^{-1}x}$ $\therefore \log u = \log x^{\sin^{-1}x}$ $\therefore \log u = \sin^{-1}x \log x$ $\therefore \frac{1}{u}\frac{du}{dx} = \sin^{-1}x\left(\frac{1}{x}\right) + \log x\left(\frac{1}{\sqrt{1-x^2}}\right)$ $\therefore \frac{1}{u}\frac{du}{dx} = \frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}$ $\therefore \frac{du}{dx} = u\left[\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right]$ $\therefore \frac{du}{dx} = x^{\sin^{-1}x}\left[\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right]$ $v = \sin^{-1}x$ $\therefore \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{x^{\sin^{-1}x}\left[\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right]}{\frac{1}{\sqrt{1-x^2}}}$	04 ½ 1 ½ 1 1

<table>
<tr><th>Q. No.</th><th>Sub Q. N.</th><th>Answer</th><th>Marking Scheme</th></tr>
<tr><td>4</td><td>d)</td><td>$\therefore \frac{dy}{dz} = x^{\sin^{-1} x}\left(\sqrt{1-x^2}\right)\left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right]$</td><td></td></tr>
<tr><td></td><td>e)
Ans</td><td>If $xy = \log(xy)$ show that $\frac{dy}{dx} = -\frac{y}{x}$
$xy = \log(xy)$
$\therefore x\frac{dy}{dx} + y(1) = \frac{1}{xy}\left(x\frac{dy}{dx} + y(1)\right)$
$\therefore x\frac{dy}{dx} + y = \frac{1}{y}\frac{dy}{dx} + \frac{1}{x}$
$\therefore x\frac{dy}{dx} - \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} - y$
$\therefore \left(x - \frac{1}{y}\right)\frac{dy}{dx} = \frac{1}{x} - y$
$\therefore \left(\frac{xy-1}{y}\right)\frac{dy}{dx} = \frac{1-xy}{x}$
$\therefore \frac{dy}{dx} = \frac{-(xy-1)}{x} \times \frac{y}{xy-1}$
$\therefore \frac{dy}{dx} = -\frac{y}{x}$</td><td>**04**
1
½
1
½
1</td></tr>
<tr><td></td><td>f)
Ans</td><td>If u and v are differentiable functions of x and $y = u + v$ then prove that:
$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$
Given $y = u + v$
Let $\delta u, \delta v, \delta y$ are small increments in u, v, y respectively corresponding to increment δx in x.
$\therefore y + \delta y = u + \delta u + v + \delta v$
$\therefore \delta y = u + \delta u + v + \delta v - y$
$\therefore \delta y = u + \delta u + v + \delta v - (u + v)$
$\therefore \delta y = u + \delta u + v + \delta v - u - v$
$\therefore \delta y = \delta u + \delta v$
$\therefore \frac{\delta y}{\delta x} = \frac{\delta u + \delta v}{\delta x}$
$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{\delta u}{\delta x} + \lim_{\delta x \to 0} \frac{\delta v}{\delta x}$</td><td>**04**
1
1
1</td></tr>
</table>

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4	f)	$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$	1
		--	
5		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	Evaluate: $\lim_{x \to 0} \frac{\log(2+x) - \log(2-x)}{x}$	**04**
	Ans	$\lim_{x \to 0} \frac{\log(2+x) - \log(2-x)}{x}$	
		$= \lim_{x \to 0} \frac{1}{x} \log\left(\frac{2+x}{2-x}\right)$	½
		$= \lim_{x \to 0} \frac{1}{x} \log\left(\frac{1+\frac{x}{2}}{1-\frac{x}{2}}\right)$	
		$= \lim_{x \to 0} \log\left(\frac{1+\frac{x}{2}}{1-\frac{x}{2}}\right)^{\frac{1}{x}}$	½
		$= \log\left[\frac{\left(\lim_{x \to 0}\left(1+\frac{x}{2}\right)^{\frac{2}{x}}\right)^{\frac{1}{2}}}{\left(\lim_{x \to 0}\left(1-\frac{x}{2}\right)^{-\frac{2}{x}}\right)^{-\frac{1}{2}}}\right]$	1
		$= \log\left[\frac{e^{\frac{1}{2}}}{e^{-\frac{1}{2}}}\right]$	1
		$= \log(e)^{\frac{1}{2}+\frac{1}{2}}$ $= \log e$ $= 1$	1
		--	
	b)	Show that the roots of the equation $x^3 - 9x + 1 = 0$ lies between 2 and 3. Obtain the roots by Bisection method (3 iterations only)	**04**
	Ans	Let $f(x) = x^3 - 9x + 1$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	b)	$f(2) = -9 < 0$ $f(3) = 1 > 0$ $\therefore$ root lies in $(2,3)$ $x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$ $f(x_1) = -5.875 < 0$ the root lies in $(2.5, 3)$ $x_2 = \frac{a+x_1}{2} = \frac{2.5+3}{2} = 2.75$ $f(x_2) = -2.953 < 0$ the root lies in $(2.75, 3)$ $x_3 = \frac{a+x_2}{2} = \frac{2.75+3}{2} = 2.875$	1 1 1 1
		OR Let $f(x) = x^3 - 9x + 1$ $f(2) = -9 < 0$ $f(3) = 1 > 0$ $\therefore$ root lies in $(2,3)$	1

Iterations	a	b	$x = \frac{a+b}{2}$	$f(x)$
I	2	3	2.5	-5.875
II	2.5	3	2.75	-2.953
III	2.75	3	2.875	---

Marking: 1+1+1

Q. No.	Sub Q. N.	Answer	Marking Scheme
	c)	Use Newton-Raphson method , Evaluate : $\sqrt[3]{100}$ (Up to three iterations only)	**04**
	Ans	Let $x = \sqrt[3]{100}$ $\therefore x^3 = 100$ $\therefore x^3 - 100 = 0$ $\therefore f(x) = x^3 - 100$ $f(4) = -36 < 0$ $f(5) = 25 > 0$ $f'(x) = 3x^2$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	c)	Initial root $x_0 = 5$	
		$\therefore f'(5) = 75$	
		$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 4.667$	1
		$x_2 = 4.667 - \frac{f(4.667)}{f'(4.667)} = 4.642$	1
		$x_3 = 4.642 - \frac{f(4.642)}{f'(4.642)} = 4.642$	1
		OR	
		Let $x = \sqrt[3]{100}$	
		$\therefore x^3 = 100$	
		$\therefore x^3 - 100 = 0$	
		$\therefore f(x) = x^3 - 100$	
		$f(4) = -36 < 0$	1
		$f(5) = 25 > 0$	½
		$f'(x) = 3x^2$	
		Initial root $x_0 = 5$	
		$x_i = x - \frac{f(x)}{f'(x)}$	
		$x_i = x - \frac{x^3 - 100}{3x^2}$	
		$x_i = \frac{3x^3 - x^3 + 100}{3x^2}$	
		$x_i = \frac{2x^3 + 100}{3x^2}$	1
		$x_1 = 4.667$	½
		$x_2 = 4.642$	½
		$x_3 = 4.642$	½
		--	
	d)	Using Regula-Falsi method , find the root of $xe^x - 3 = 0$ (three iterations only)	**04**
	Ans	Let $f(x) = xe^x - 3$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	d)	$f(1) = -0.282 < 0$	
		$f(2) = 11.778 > 0$	1
		$\therefore$ the root lies in $(1,2)$	
		$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{1(11.778)-2(-0.282)}{11.778+0.282} = 1.023$	1
		$f(x_1) = -0.154 < 0$	
		the root lies in $(1.023,2)$	
		$x_2 = \frac{1.023(11.778)-2(-0.154)}{11.778+0.154} = 1.036$	1
		$f(x_2) = -0.081 < 0$	
		the root lies in $(1.036,2)$	
		$x_3 = \frac{1.036(11.778)-2(-0.081)}{11.778+0.081} = 1.043$	1
		OR	
		Let $f(x) = xe^x - 3$	
		$f(1) = -0.282 < 0$	
		$f(2) = 11.778 > 0$	1
		$\therefore$ the root lies in $(1,2)$	

Iterations	a	b	$f(a)$	$f(b)$	$x = \frac{af(b)-bf(a)}{f(b)-f(a)}$	$f(x)$
I	1	2	– 0.282	11.778	1.023	– 0.154
II	1.023	2	– 0.154	11.778	1.036	– 0.081
III	1.036	2	– 0.081	11.778	1.043	– – –

Marking scheme: 1+1+1

Q. No.	Sub Q. N.	Answer	Marking Scheme
	e)	Using bisection method, find the approximate root of $x^3 - 2x - 5 = 0$ in the interval $(2,3)$ (3 iterations only)	**04**
	Ans	Let $f(x) = x^3 - 2x - 5$	
		$f(2) = -1 < 0$	
		$f(3) = 16 > 0$	1
		$\therefore$ root lies in $(2,3)$	
		$x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$	1
		$f(x_1) = 5.625 > 0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5	e)	the root lies in $(2, 2.5)$	
		$x_2 = \frac{2+2.5}{2} = 2.25$	1
		$f(x_2) = 1.891 > 0$	
		the root lies in $(2, 2.25)$	
		$x_3 = \frac{2+2.25}{2} = 2.125$	1
		OR	
		Let $f(x) = x^3 - 2x - 5$	
		$f(2) = -1 < 0$	1
		$f(3) = 16 > 0$	
		$\therefore$ root lies in $(2,3)$	

Iterations	a	b	$x = \frac{a+b}{2}$	$f(x)$
I	2	3	2.5	5.625
II	2	2.5	2.25	1.891
III	2	2.25	2.125	---

Marking scheme: 1+1+1

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f) Find the root of the equation using Newton-Raphson method $x^2 - 4x - 6 = 0$ near to 5.(three iterations only) — **04**

Ans

Let $f(x) = x^2 - 4x - 6$

$f(5) = -1 < 0$

$f'(x) = 2x - 4$

$\therefore f'(5) = 6$ — 1

Initial root $x_0 = 5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 5.167$$ — 1

$$x_2 = 5.167 - \frac{f(5.167)}{f'(5.167)} = 5.162$$ — 1

$$x_2 = 5.162 - \frac{f(5.162)}{f'(5.162)} = 5.162$$ — 1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5	f)	OR	
		Let $f(x) = x^2 - 4x - 6$	
		$f(5) = -1 < 0$	
		$f(6) = 6 > 0$	1
		$f'(x) = 2x - 4$	½
		Initial root $x_0 = 5$	
		$\therefore f'(5) = -1$	
		$x_i = x - \dfrac{f(x)}{f'(x)}$	
		$x_i = x - \dfrac{x^2 - 4x - 6}{2x - 4}$	
		$x_i = \dfrac{2x^2 - 4x - x^2 + 4x + 6}{2x - 4}$	
		$x_i = \dfrac{x^2 + 6}{2x - 4}$	1
		$\therefore f'(5) = -1$	
		$x_1 = 5.167$	½
		$x_2 = 5.162$	½
		$x_3 = 5.162$	½
		--	
6		**Solve any FOUR of the following:**	**16**
	a)	Solve the following equations by Gauss elimination method $x + y + z = 6$, $3x - y + 3z = 10$, $5x + 5y - 4z = 3$	**04**
	Ans	$x + y + z = 6$ $3x - y + 3z = 10$ $5x + 5y - 4z = 3$	
		$x + y + z = 6$ and $5x + 5y + 5z = 30$ $3x - y + 3z = 10$ and $5x + 5y - 4z = 3$ (adding) and (subtracting)	
		$4x + 4z = 16$ and $9z = 27$	½ +½
		$\therefore x + z = 4$ and $\therefore z = 3$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6	a)	$\therefore\ x = 1$	1
		$y = 2$	1
		$z = 3$	1
		Note: *In the above solution, first y is eliminated and then x is eliminated to find the value of z first. If in case the problem is solved by elimination of another unknown i. e., either first x or z , appropriate marks to be given as per above scheme of marking.*	
	b)	By using first principle , prove that $\frac{d}{dx}(\sin x) = \cos x$	**04**
	Ans	$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	
		$\frac{dy}{dx} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$	1
		$\frac{dy}{dx} = \lim_{h \to 0} \frac{2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}$	1
		$\frac{dy}{dx} = 2\lim_{h \to 0} \frac{\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$	
		$\frac{dy}{dx} = 2\left(\lim_{h \to 0} \cos\left(\frac{2x+h}{2}\right)\right)\left(\lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} . \frac{1}{2}\right)$	1
		$\frac{dy}{dx} = 2(\cos x)\frac{1}{2}$	
		$\frac{dy}{dx} = \cos x$	1
	c)	Solve by Jacobi's method upto 3 iterations only:	**04**
		$30x + y + z = 32 \quad , \quad x + 30y + z = 32 \quad , \quad x + y + 30z = 32$	
	Ans	$x = \frac{1}{30}(32 - y - z)$	
		$y = \frac{1}{30}(32 - x - z)$	1
		$z = \frac{1}{30}(32 - x - y)$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6	c)	Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 1.067$ $y_1 = 1.067$ $z_1 = 1.067$	1
		$x_2 = 0.996$ $y_2 = 0.996$ $z_2 = 0.996$	1
		$x_3 = 1$ $y_3 = 1$ $z_3 = 1$	1
		--	
	d)	Solve by Gauss-Seidal method (3 iterations only) $6x + y + z = 105$, $4x + 8y + 3z = 155$, $5x + 4y - 10z = 65$	**04**
	Ans	$x = \frac{1}{6}(105 - y - z)$ $y = \frac{1}{8}(155 - 4x - 3z)$ $z = \frac{1}{-10}(65 - 5x - 4y)$	1
		Starting with $y_0 = z_0 = 0$ $x_1 = 17.5$ $y_1 = 10.625$ $z_1 = 6.5$	1
		$x_2 = 14.646$ $y_2 = 9.615$ $z_2 = 4.669$	1
		$x_3 = 15.119$ $y_3 = 10.065$ $z_3 = 5.086$	1
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	e)	Solve by Gauss elimination method	04
		$x+y+z=4\quad,\quad 2x+y+z=5\quad,\quad 3x+2y+z=7$	
	Ans	$x+y+z=4$ $2x+y+z=5$ $3x+2y+z=7$	
		$x+y+z=4$ and $2x+2y+2z=8$ $2x+y+z=5$ and $3x+2y+z=7$ $-$ ___________ $-$ ___________ $-x=-1$ $\quad -x+z=1$	½ +½
		$\therefore\ x=1$	1
		$y=1$	1
		$z=2$	1
		Note: *In the above solution, first y is eliminated and then z is eliminated to find the value of x first. If in case the problem is solved by elimination of another unknown i. e., either first x or z, appropriate marks to be given as per above scheme of marking.*	
		--	
	f)	Solve by Jacobi's method	04
		$4x+y+2z=12\quad,\quad -x+11y+4z=33\quad,\quad 2x-3y+8z=20\quad$ (3 iterations only)	
	Ans	$x=\frac{1}{4}(12-y-2z)$ $y=\frac{1}{11}(33+x-4z)$ $z=\frac{1}{8}(20-2x+3y)$	1
		Starting with $x_0=y_0=z_0=0$	
		$x_1=3$ $y_1=3$ $z_1=2.5$	1
		$x_2=1$ $y_2=2.364$ $z_2=2.875$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6	f)	$x_3 = 0.972$ $y_3 = 2.045$ $z_3 = 3.137$ -- ***Important Note*** *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* -- --	1