WINTER –2017 EXAMINATION

Model Answer

Subject Code: **17216**

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance.(Not applicable for subject English and Communication Skills)
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
Q. 1		**Attempt any TEN of the following:**	**20**
	a)	Find x and y if $x(1-i)+y(2+i)+6=0$	**02**
	Ans	$x(1-i)+y(2+i)+6=0$	
		$(x+2y+6)+i(-x+y)=0$	½
		$\therefore x+2y+6=0$	
		$-x+y=0$	
		$x+2y=-6$	½
		$-x+y=0$	
		$3y=-6$	
		$\Rightarrow \quad y=-2$	½
		$x=-2$	½
		--	
	b)	Express in $a+ib$ form $\dfrac{2-\sqrt{3}\,i}{1+i}$	**02**
	Ans	$\dfrac{2-\sqrt{3}\,i}{1+i}=\dfrac{2-\sqrt{3}\,i}{1+i}\times\dfrac{1-i}{1-i}$	½
		$=\dfrac{2-\sqrt{3}\,i-2i+\sqrt{3}i^2}{1-i^2}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	b)	$=\dfrac{\left(2-\sqrt{3}\right)+i\left(-2-\sqrt{3}\right)}{1+1}$	½
		$=\left(\dfrac{2-\sqrt{3}}{2}\right)+i\left(\dfrac{-2-\sqrt{3}}{2}\right)$	½
		--	
	c)	If $f\left(x\right)=x^2-2x-5$ and $t=y-2$, find $f\left(t\right)$	**02**
	Ans	$f\left(x\right)=x^2-2x-5$,	
		and $t=y-2$	
		$\therefore\ f\left(t\right)=t^2-2t-5$	
		$=\left(y-2\right)^2-2\left(y-2\right)-5$	½
		$=\ y^2-4y+4-2y+4-5$	1
		$=y^2-6y+3$	½
		--	
	d)	If $f\left(x\right)=\log_a x$, prove that $f\left(m\right)+f\left(n\right)=f\left(m.n\right)$	**02**
	Ans	$f\left(x\right)=\log_a x$	
		$\therefore\ f\left(m\right)=\log_a m$	½
		$\therefore\ f\left(n\right)=\log_a n$	½
		$f\left(m\right)+f\left(n\right)=\log_a m+\log_a n$	
		$=\log_a\left(m.n\right)$	½
		$=\ f\left(m.n\right)$	½
		--	
	e)	Evaluate: $\lim\limits_{x\to -4}\dfrac{x^2+3x-4}{x^2+7x+12}$	**02**
	Ans	$\lim\limits_{x\to -4}\dfrac{x^2+3x-4}{x^2+7x+12}$	
		$=\lim\limits_{x\to -4}\dfrac{\left(x-1\right)\left(x+4\right)}{\left(x+3\right)\left(x+4\right)}$	1
		$=\lim\limits_{x\to -4}\dfrac{\left(x-1\right)}{\left(x+3\right)}$	
		$=\dfrac{\left(-4-1\right)}{\left(-4+3\right)}=\dfrac{-5}{-1}=5$	1
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	Evaluate: $\lim_{x\to 0}\frac{4x-\tan x}{3x+\tan x}$	**02**
	Ans	$\lim_{x\to 0}\frac{4x-\tan x}{3x+\tan x}$	
		$=\lim_{x\to 0}\frac{\frac{4x-\tan x}{x}}{\frac{3x+\tan x}{x}}$	
		$=\frac{4-\lim_{x\to 0}\frac{\tan x}{x}}{3+\lim_{x\to 0}\frac{\tan x}{x}}$	1
		$=\frac{4-1}{3+1}$	
		$=\frac{3}{4}$	1
		--	
	g)	Evaluate: $\lim_{x\to 0}\left(\frac{e^{3x}-1}{4x}\right)$	**02**
	Ans	$\lim_{x\to 0}\left(\frac{e^{3x}-1}{4x}\right)$	
		$=\lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\times\frac{3}{4}\right)$	1
		$=\left(1\times\frac{3}{4}\right)$	
		$=\frac{3}{4}$	1
		--	
	h)	Find $\frac{dy}{dx}$, if $y=\log\left[\tan\left(4-3x\right)\right]$	**02**
	Ans	$y=\log\left[\tan\left(4-3x\right)\right]$	
		$\frac{dy}{dx}=\frac{1}{\tan\left(4-3x\right)}\sec^2\left(4-3x\right)\left(-3\right)$ OR $\frac{dy}{dx}=\frac{1}{\tan\left(4-3x\right)}\frac{d}{dx}\left[\tan\left(4-3x\right)\right]$	½
		$\frac{dy}{dx}=\frac{-3\sec^2\left(4-3x\right)}{\tan\left(4-3x\right)}$ $\frac{dy}{dx}=\frac{\sec^2\left(4-3x\right)}{\tan\left(4-3x\right)}\frac{d}{dx}\left(4-3x\right)$	½
		$\frac{dy}{dx}=-3\cot\left(4-3x\right)\sec^2\left(4-3x\right)$ $\frac{dy}{dx}=-3\cot\left(4-3x\right)\sec^2\left(4-3x\right)$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
1	h)	OR	
		$y=\log\left[\tan\left(4-3x\right)\right]$	
		$\frac{dy}{dx}=\frac{1}{\tan\left(4-3x\right)}\sec^2\left(4-3x\right)\left(-3\right)$	1
		$\frac{dy}{dx}=-3\frac{\cos\left(4-3x\right)}{\sin\left(4-3x\right)}\frac{1}{\cos^2\left(4-3x\right)}$	½
		$\frac{dy}{dx}=-3\operatorname{cosec}\left(4-3x\right)\sec\left(4-3x\right)$	½
		----------	**02**
	i)	Find $\frac{dy}{dx}$, if $x=a\left(\theta-\sin\theta\right), y=a\left(1-\cos\theta\right)$	
	Ans	$x=a\left(\theta-\sin\theta\right), y=a\left(1-\cos\theta\right)$	
		$\frac{dx}{d\theta}=a\left(1-\cos\theta\right)$ and $\frac{dy}{d\theta}=a\sin\theta$	½+½
		$\because \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\sin\theta}{a\left(1-\cos\theta\right)}$	½
		$\frac{dy}{dx}=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$	
		$\frac{dy}{dx}=\cot\frac{\theta}{2}$	½
		----------	
	j)	Differentiate $\cos^{-1}\left(1-2\sin^2 x\right)$	**02**
	Ans	Let $y=\cos^{-1}\left(1-2\sin^2 x\right)$	
		$y=\cos^{-1}\left(\cos 2x\right)$	1
		$y=2x$	
		$\frac{dy}{dx}=2$	1
		----------	
	k)	Show that there exist a root of the equation $x^3+2x^2-8=0$ between 1 and 2.	**02**
	Ans	Let $f\left(x\right)=x^3+2x^2-8$	
		$f\left(1\right)=-5<0$	1
		$f\left(2\right)=8>0$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	k)	$\therefore$ root lies between 1 and 2	
		--	
	l)	Solve the f1ollowing equations by using Gauss-Seidal method $\left(\text{only first iteration}\right)$	02
		$10x+2y+z=9; x+10y-z=-22; -2x+3y+10z=22$	
	Ans	$x=\dfrac{9-2y-z}{10}$	
		$y=\dfrac{-22-x+z}{10}$	1
		$z=\dfrac{22+2x-3y}{10}$	
		Initial approximations : $x_0=y_0=z_0=0$	
		$x_1=0.9$, $y_1=-2.29$, $z_1=3.067$	1
		--	
2.		**Attempt any FOUR of the following:**	20
	a)	Simplify using De-Moiver's theorem	
		$\dfrac{\left(\cos\theta-i\sin\theta\right)^6\left(\cos5\theta-i\sin5\theta\right)^{-2}}{\left(\cos8\theta+i\sin8\theta\right)\frac{1}{2}}$	04
	Ans	$\dfrac{\left(\cos\theta-i\sin\theta\right)^6\left(\cos5\theta-i\sin5\theta\right)^{-2}}{\left(\cos8\theta+i\sin8\theta\right)\frac{1}{2}}$	
		$=\dfrac{2\left(\cos\theta+i\sin\theta\right)^{-6}\left(\cos\theta+i\sin\theta\right)^{10}}{\left(\cos\theta+i\sin\theta\right)^{8}}$	2
		$=2\left(\cos\theta+i\sin\theta\right)^{-6+10-8}$	1
		$=2\left(\cos\theta+i\sin\theta\right)^{-4}$	½
		$=2\left(\cos4\theta-i\sin4\theta\right)$	½
		--	
	b)	Find cube root of unity.	
	Ans	$w=\sqrt[3]{1}$	04
		$\therefore w^3=1$	
		Put $w^3=z$	
		$\therefore z=1+0i$	
		$x=1>0, y=0$	
		$r=\lvert z\rvert=\sqrt{1+0}=1$	½

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2.	b)	$\theta=\tan^{-1}\left(\frac{0}{1}\right)=0$	½
		General polar form is, $z=r\left[\cos\left(2n\pi+\theta\right)+i\sin\left(2n\pi+\theta\right)\right]$	
		$w^3=1\left(\cos 2n\pi+i\sin 2n\pi\right)$	½
		$w=\left(\cos 2n\pi+i\sin 2n\pi\right)^{\frac{1}{3}}$	
		$w=\cos\left(\frac{2n\pi}{3}\right)+i\sin\left(\frac{2n\pi}{3}\right)$; n = 0,1,2	1
		when $n=0$	
		$w_1=\cos 0+i\sin 0=1$	½
		when $n=1$	
		$w_2=\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$	½
		when $n=2$	
		$w_3=\cos\left(\frac{4\pi}{3}\right)+i\sin\left(\frac{4\pi}{3}\right)=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$	½
		--	
	c)	If $x+iy=\sin\left(A+iB\right)$ prove that $\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}=1$	**04**
	Ans	$x+iy=\sin\left(A+iB\right)$	
		$x+iy=\sin A\cos\left(iB\right)+\cos A\sin\left(iB\right)$	1
		$=\sin A\cosh B+i\cos A\sinh B$	1
		$\therefore x=\sin A\cosh B, y=\cos A\sinh B$	
		$\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}=\frac{\sin^2 A\cosh^2 B}{\cosh^2 B}+\frac{\cos^2 A\sinh^2 B}{\sinh^2 B}$	1
		$=\sin^2 A+\cos^2 A$	
		$=1$	1
		--	
	d)	Prove that $\left(1+\cos\theta+i\sin\theta\right)^n+\left(1+\cos\theta-i\sin\theta\right)^n=2^{n+1}.\cos^n\left(\frac{\theta}{2}\right).\cos\left(\frac{n\theta}{2}\right)$	**04**
	Ans	$\left(1+\cos\theta+i\sin\theta\right)^n+\left(1+\cos\theta-i\sin\theta\right)^n$	
		$=\left(2\cos^2\frac{\theta}{2}+i2\cos\frac{\theta}{2}\sin\frac{\theta}{2}\right)^n+\left(2\cos^2\frac{\theta}{2}-i2\cos\frac{\theta}{2}\sin\frac{\theta}{2}\right)^n$	1½
		$=2^n\cos^n\frac{\theta}{2}\left[\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^n+\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)^n\right]$	½

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2.	d)	$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}+\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$	1
		$=2^n.\cos^n\frac{\theta}{2}.\left(2\cos\frac{n\theta}{2}\right)$	½
		$=2^{n+1}.\cos^n\left(\frac{\theta}{2}\right).\cos\left(\frac{n\theta}{2}\right)$	½
		---	
	e)	If $f(x)=\frac{2x+5}{3x-4}$ and $t=\frac{5+4x}{3x-2}$ show that $f(t)=x$	**04**
	Ans	$f(x)=\frac{2x+5}{3x-4}$ and $t=\frac{5+4x}{3x-2}$	
		$f(t)=\frac{2t+5}{3t-4}$	½
		$=\frac{2\left(\frac{5+4x}{3x-2}\right)+5}{3\left(\frac{5+4x}{3x-2}\right)-4}$	1
		$=\frac{2(5+4x)+5(3x-2)}{3(5+4x)-4(3x-2)}$	½
		$=\frac{10+8x+15x-10}{15+12x-12x+8}$	1
		$=\frac{23x}{23}$	
		$=x$	1
		---	
	f)	If $f(x)=\log\left(\frac{1+x}{1-x}\right)$, show that $f(a)+f(b)=f\left(\frac{a+b}{1+ab}\right)$	**04**
	Ans	$f(a)+f(b)=\log\left(\frac{1+a}{1-a}\right)+\log\left(\frac{1+b}{1-b}\right)$	1
		$=\log\left(\frac{1+a}{1-a}.\frac{1+b}{1-b}\right)$	½
		$=\log\left(\frac{1+a+b+ab}{1-a-b+ab}\right)$	½
		$f\left(\frac{a+b}{1+ab}\right)=\log\left(\frac{1+\frac{a+b}{1+ab}}{1-\frac{a+b}{1+ab}}\right)$	1

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2.	f)	$=\log\left(\dfrac{\dfrac{1+ab+a+b}{1+ab}}{\dfrac{1+ab-(a+b)}{1+ab}}\right)$	½
		$=\log\left(\dfrac{1+a+b+ab}{1-a-b+ab}\right)$	½
		$\therefore f(a)+f(b)=f\left(\dfrac{a+b}{1+ab}\right)$	
		OR	
		$f(a)+f(b)=\log\left(\dfrac{1+a}{1-a}\right)+\log\left(\dfrac{1+b}{1-b}\right)$	1
		$=\log\left(\dfrac{1+a}{1-a}.\dfrac{1+b}{1-b}\right)$	½
		$=\log\left(\dfrac{1+a+b+ab}{1-a-b+ab}\right)$	½
		$=\log\left(\dfrac{1+ab+a+b}{1+ab-(a+b)}\right)$	
		$=\log\left(\dfrac{1+\left(\dfrac{a+b}{1+ab}\right)}{1-\left(\dfrac{a+b}{1+ab}\right)}\right)$	1
		$=f\left(\dfrac{a+b}{1+ab}\right)$	1
3.		**Attempt any FOUR of the following:**	**20**
	a)	If $f(x)=\log\left(\dfrac{x}{x-1}\right)$ show that $f(a+1)+f(a)=\log\left(\dfrac{a+1}{a-1}\right)$	**04**
	Ans	$f(a+1)+f(a)=\log\left(\dfrac{a+1}{a+1-1}\right)+\log\left(\dfrac{a}{a-1}\right)$	1+1
		$=\log\left(\dfrac{a+1}{a}\right)+\log\left(\dfrac{a}{a-1}\right)=\log\left(\dfrac{a+1}{a}.\dfrac{a}{a-1}\right)$	1
		$=\log\left(\dfrac{a+1}{a-1}\right)$	1
	b)	If $f(x)=x-\dfrac{1}{x}$, then show that $\left[f(x)\right]^3=$ f$\left(x^3\right)+3f\left(\dfrac{1}{x}\right)$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$f(x)=x-\frac{1}{x}$	
		$\therefore\ f(x^3)=x^3-\frac{1}{x^3},\ f\left(\frac{1}{x}\right)=\frac{1}{x}-x$	½+½
		$\left[f(x)\right]^3=\left(x-\frac{1}{x}\right)^3$ OR LHS $=\left[f(x)\right]^3=\left(x-\frac{1}{x}\right)^3$	½
		$=x^3-3(x)^2\left(\frac{1}{x}\right)+3(x)\left(\frac{1}{x}\right)^2-\left(\frac{1}{x}\right)^3$ $\quad =x^3-3(x)^2\left(\frac{1}{x}\right)+3(x)\left(\frac{1}{x}\right)^2-\left(\frac{1}{x}\right)^3$	1
		$=x^3-3x^2\frac{1}{x}+3x\frac{1}{x^2}-\frac{1}{x^3}$ $\quad =x^3-\frac{1}{x^3}+3\left(\frac{1}{x}-x\right)$	½
		$=x^3-\frac{1}{x^3}+3\left(\frac{1}{x}-x\right)$ RHS $=f(x^3)+3f\left(\frac{1}{x}\right)=x^3-\frac{1}{x^3}+3\left(\frac{1}{x}-x\right)$	½
		$=f(x^3)+3f\left(\frac{1}{x}\right)$ $\quad \left[f(x)\right]^3=f(x^3)+3f\left(\frac{1}{x}\right)$	½
		---	
	c)	Evaluate $\lim_{x\to0}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\right)$	**04**
	Ans	$\lim_{x\to0}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\right)$	
		$=\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\times\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$	1
		$=\lim_{x\to0}\frac{1+x-(1-x)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	1
		$=\lim_{x\to0}\frac{2x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	½
		$=\frac{2}{\sqrt{1+0}+\sqrt{1-0}}$	½
		$=1$	1
		---	
	d)	Evaluate: $\lim_{x\to\frac{\pi}{4}}\left(\frac{2-\sec^2 x}{1-\tan x}\right)$	**04**
	Ans	$\lim_{x\to\frac{\pi}{4}}\left(\frac{2-\sec^2 x}{1-\tan x}\right)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	d)	$=\lim_{x\to\frac{\pi}{4}}\left[\frac{2-\left(1+\tan^2 x\right)}{1-\tan x}\right]$	1
		$=\lim_{x\to\frac{\pi}{4}}\left(\frac{1-\tan^2 x}{1-\tan x}\right)$	½
		$=\lim_{x\to\frac{\pi}{4}}\frac{(1-\tan x)(1+\tan x)}{1-\tan x}$	1
		$=\lim_{x\to\frac{\pi}{4}}(1+\tan x)$	
		$=1+\tan\frac{\pi}{4}$	½
		$=2$	1
		--	
	e)	Evaluate $\lim_{x\to 0}\frac{6^x-3^x-2^x+1}{x^2}$	**04**
	Ans	$\lim_{x\to 0}\frac{6^x-3^x-2^x+1}{x^2}$	
		$=\lim_{x\to 0}\frac{3^x 2^x-3^x-2^x+1}{x^2}$	½
		$=\lim_{x\to 0}\frac{3^x\left(2^x-1\right)-\left(2^x-1\right)}{x^2}$	1
		$=\lim_{x\to 0}\frac{\left(3^x-1\right)\left(2^x-1\right)}{x^2}$	1
		$=\left(\lim_{x\to 0}\frac{3^x-1}{x}\right)\left(\lim_{x\to 0}\frac{2^x-1}{x}\right)$	½
		$=(\log 3)(\log 2)$	1
		--	
	f)	Evaluate $\lim_{x\to 5}\left(\frac{\log x-\log 5}{x-5}\right)$	**04**
	Ans	$\lim_{x\to 5}\left(\frac{\log x-\log 5}{x-5}\right)$	
		Put $x=5+h$ as $x\to 5$,$h\to 0$	
		$=\lim_{h\to 0}\frac{\log(5+h)-\log 5}{5+h-5}$	½
		$=\lim_{h\to 0}\frac{\log\left(\frac{5+h}{5}\right)}{h}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	f)	$=\lim_{h\to0}\frac{1}{h}\log\left(1+\frac{h}{5}\right)$	½
		$=\lim_{h\to0}\log\left(1+\frac{h}{5}\right)^{\frac{1}{h}}$	½
		$=\log\left[\lim_{h\to0}\left(1+\frac{h}{5}\right)^{\frac{5}{h}}\right]^{\frac{1}{5}}$	½
		$=\log e^{\frac{1}{5}}$	½
		$=\frac{1}{5}\log e$	
		$=\frac{1}{5}$	½
		--	
4.		**Attempt any FOUR of the following:**	**20**
	a)	Using first principal find the derivative of $\sin x$	**04**
	Ans	$\frac{dy}{dx}=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$	
		$\frac{dy}{dx}=\lim_{h\to0}\frac{\sin(x+h)-\sin x}{h}$	1
		$\frac{dy}{dx}=\lim_{h\to0}\frac{2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}$	1
		$\frac{dy}{dx}=2\lim_{h\to0}\frac{\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$	½
		$\frac{dy}{dx}=2\left(\lim_{h\to0}\cos\left(\frac{2x+h}{2}\right)\right)\left(\lim_{h\to0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\cdot\frac{1}{2}\right)$	1
		$\frac{dy}{dx}=2(\cos x)\frac{1}{2}$	
		$\frac{dy}{dx}=\cos x$	½
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4.	b)	Find $\frac{dy}{dx}$ if $x=a(\cos\theta+\theta\sin\theta)$ and $y=a\left(\sin\theta-\theta\cos\theta\right)$	**04**
	Ans	$x=a(\cos\theta+\theta\sin\theta)$, $y=a\left(\sin\theta-\theta\cos\theta\right)$	
		$\frac{dx}{d\theta}=a\left(-\sin\theta+\theta\cos\theta+\sin\theta\right)=a\theta\cos\theta$	1
		$\frac{dy}{d\theta}=a\left(\cos\theta+\theta\sin\theta-\cos\theta\right)=a\theta\sin\theta$	1
		$\therefore\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$\therefore\frac{dy}{dx}=\frac{a\theta\sin\theta}{a\theta\cos\theta}$	1
		$\therefore\frac{dy}{dx}=\tan\theta$	1
		--	
	c)	Find $\frac{dy}{dx}$ if $y=\sin^{-1}\left[\frac{\cos x+\sin x}{\sqrt{2}}\right]$	**04**
	Ans	$y=\sin^{-1}\left(\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right)$	½
		$y=\sin^{-1}\left(\sin\frac{\pi}{4}\cos x+\cos\frac{\pi}{4}\sin x\right)$	1
		$y=\sin^{-1}\left[\sin\left(\frac{\pi}{4}+x\right)\right]$	1
		$y=\frac{\pi}{4}+x$	½
		$\frac{dy}{dx}=0+1=1$	1
		OR	
		$y=\sin^{-1}\left(\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right)$	½
		$y=\sin^{-1}\left(\cos\frac{\pi}{4}\cos x+\sin\frac{\pi}{4}\sin x\right)$	½
		$y=\sin^{-1}\left[\cos\left(\frac{\pi}{4}-x\right)\right]$	1
		$y=\sin^{-1}\left[\sin\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-x\right)\right)\right]$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	c)	$y=\frac{\pi}{4}+x$	½
		$\frac{dy}{dx}=0+1=1$	1
		--	
	d)	If $e^{y}=y^{x}$, prove that $\frac{dy}{dx}=\frac{(\log y)^{2}}{(\log y-1)}$	**04**
	Ans	$e^{y}=y^{x}\Rightarrow y\log e=x\log y$	½
		$y=x\log y$	
		$\frac{dy}{dx}=x\frac{1}{y}\frac{dy}{dx}+\log y(1)$	1
		$\frac{dy}{dx}\left(1-\frac{x}{y}\right)=\log y$	½
		$\frac{dy}{dx}\left(1-\frac{x}{x\log y}\right)=\log y$ OR $\frac{dy}{dx}\left(\frac{y-x}{y}\right)=\log y$	
		$\frac{dy}{dx}\left(1-\frac{1}{\log y}\right)=\log y$; $\frac{dy}{dx}=\frac{y\log y}{y-x}$	½
		$\frac{dy}{dx}\left(\frac{\log y-1}{\log y}\right)=\log y$; $\frac{dy}{dx}=\frac{(x\log y)\log y}{x\log y-x}$	½
		$\frac{dy}{dx}=\frac{(\log y)^{2}}{(\log y-1)}$; $\frac{dy}{dx}=\frac{(\log y)^{2}}{(\log y-1)}$	1
		--	
	e)	If $y=(\sin x)^{\log x}$, find $\frac{dy}{dx}$.	**04**
	Ans	$y=(\sin x)^{\log x}$	
		$\log y=\log x.\log(\sin x)$	½
		$\frac{1}{y}\frac{dy}{dx}=\log x.\frac{1}{\sin x}.\cos x+\log(\sin x).\frac{1}{x}$	1½
		$\frac{dy}{dx}=y\left(\log x\cot x+\frac{1}{x}\log(\sin x)\right)$	1
		$\frac{dy}{dx}=(\sin x)^{\log x}\left(\log x\cot x+\frac{1}{x}\log(\sin x)\right)$	1
		--	
	f)	If $x^{3}+y^{3}=3axy$, find $\frac{dy}{dx}$ at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$.	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	f)	$x^3 + y^3 = 3axy$	
	Ans	$3x^2 + 3y^2 \frac{dy}{dx} = 3a\left(x\frac{dy}{dx} + y(1)\right)$	1
		$3y^2 \frac{dy}{dx} - 3ax\frac{dy}{dx} = 3ay - 3x^2$	½
		$\frac{dy}{dx} = \frac{3ay - 3x^2}{3y^2 - 3ax}$	½
		$\frac{dy}{dx}_{\left(\frac{3a}{2},\frac{3a}{2}\right)} = \frac{3a\left(\frac{3a}{2}\right) - 3\left(\frac{3a}{2}\right)^2}{3\left(\frac{3a}{2}\right)^2 - 3a\left(\frac{3a}{2}\right)}$	1
		$\frac{dy}{dx}_{\left(\frac{3a}{2},\frac{3a}{2}\right)} = -1$	1
5.		**Attempt any FOUR of the following:**	**20**
	a)	Evaluate $\lim_{x\to\infty}\left(\frac{1+3x}{3x-2}\right)^{2x}$	**04**
	Ans	$\lim_{x\to\infty}\left(\frac{1+3x}{3x-2}\right)^{2x}$	
		$= \lim_{x\to\infty}\left(\frac{\frac{1+3x}{3x}}{\frac{3x-2}{3x}}\right)^{2x}$	½
		$= \lim_{x\to\infty}\left(\frac{1+\frac{1}{3x}}{1-\frac{2}{3x}}\right)^{2x}$	1
		$= \frac{\lim_{x\to\infty}\left[\left(1+\frac{1}{3x}\right)^{3x}\right]^{\frac{2}{3}}}{\lim_{x\to\infty}\left[\left(1-\frac{2}{3x}\right)^{\frac{3x}{-2}}\right]^{\frac{-4}{3}}}$	1½
		$= \frac{e^{\frac{2}{3}}}{e^{-\frac{4}{3}}} = e^2$	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)	Evaluate $\lim_{x\to a}\left(\frac{\cos x-\cos a}{\sqrt{x}-\sqrt{a}}\right)$	**04**
	Ans	$\lim_{x\to a}\left(\frac{\cos x-\cos a}{\sqrt{x}-\sqrt{a}}\right)=\lim_{x\to a}\left(\frac{\cos x-\cos a}{\sqrt{x}-\sqrt{a}}\times\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}\right)$	½
		$=\lim_{x\to a}\left(\frac{\cos x-\cos a}{x-a}\right)\left(\sqrt{x}+\sqrt{a}\right)$	½
		put $x=a+h,$ as $x\to a, h\to 0$	
		$=\lim_{h\to 0}\left(\frac{\cos(a+h)-\cos a}{a+h-a}\right).\lim_{h\to 0}\left(\sqrt{a+h}+\sqrt{a}\right)$	½
		$=\left(\sqrt{a+0}+\sqrt{a}\right)\lim_{h\to 0}\left(\frac{-2\sin\left(\frac{a+h+a}{2}\right)\sin\left(\frac{a+h-a}{2}\right)}{h}\right)$	1
		$=\left(2\sqrt{a}\right)\lim_{h\to 0}\left(\frac{-2\sin\left(a+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}\right)$	
		$=\left(-4\sqrt{a}\right)\left(\lim_{h\to 0}\sin\left(a+\frac{h}{2}\right)\right).\lim_{h\to 0}\left(\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\frac{1}{2}\right)$	1
		$=-4\sqrt{a}\sin(a+0)\left(1\times\frac{1}{2}\right)$	
		$=-2\sqrt{a}\sin a$	½
		--	
	c)	Using Bisection method find the approximate root of $x^3-x-4=0$ (Three iterations only).	**04**
	Ans	Let $f(x)=x^3-x-4$	
		$f(1)=-4<0$	
		$f(2)=2>0$	
		$\therefore$ root lies in $(1,2)$	1
		$x_1=\frac{a+b}{2}=\frac{1+2}{2}=1.5$	1
		$f(1.5)=-2.125<0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$\therefore$ the root lies in (1.5,2)	
		$x_2 = \frac{x_1 + b}{2} = \frac{1.5+2}{2} = 1.75$	1
		$f(x_2) = -0.39 < 0$	
		$\therefore$ the root lies in (1.75,2)	
		$x_3 = \frac{x_2 + b}{2} = \frac{1.75+2}{2} = 1.875$	1
		OR	
		Let $f(x) = x^3 - x - 4$	
		$f(1) = -4 < 0$	
		$f(2) = 2 > 0$	
		$\therefore$ root lies in $(1,2)$	1

a	b	$x = \frac{a+b}{2}$	$f(x)$	Marks
1	2	1.5	-2.125	1
1.5	2	1.75	-0.39	1
1.75	2	1.875	---	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
	d)	Using Regula-Falsi method, Find approximate root of $x^3 - 9x + 1 = 0$ (Three iterations only)	**04**
	Ans	Let $f(x) = x^3 - 9x + 1$	
		$f(2) = -9 < 0$	
		$f(3) = 1 > 0$	1
		$\therefore$ the root lies in $(2,3)$	
		$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2(1) - 3(-9)}{1+9} = 2.9$	1
		$f(x_1) = -0.711 < 0$	
		$\therefore$ root lies in $(2.9,3)$	
		$x_2 = \frac{2.9(1) - 3(-0.711)}{1+0.711} = 2.942$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	d)	$f(x_2) = -0.014 < 0$ the root lies in $(2.942, 3)$ $x_3 = \frac{2.942(1) - 3(-0.014)}{1 + 0.014} = 2.943$	1
		OR Let $f(x) = x^3 - 9x + 1$ $f(2) = -9 < 0$ $f(3) = 1 > 0$ $\therefore$ the root lies in $(2,3)$	1

Iterations	a	b	$f(a)$	$f(b)$	$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$	$f(x)$	Marks
I	2	3	-9	1	2.9	-0.711	1
II	2.9	3	-0.711	1	2.942	-0.014	1
III	2.942	3	-0.014	1	2.943	---	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
	e)	Solve by Newton-Raphson method $x^3 + 2x - 20 = 0$ (Three iteraions only)	04
	Ans	Let. $f(x) = x^3 + 2x - 20$ $f(2) = -8 < 0$ $f(3) = 13 > 0$	½
		$f'(x) = 3x^2 + 2$ $\therefore f'(2) = 14$	½
		$\therefore$ Initial root $x_0 = 2$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.571$	1
		$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.473$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	e)	$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.4695$	1
		OR	
		Let, $f(x) = x^3 + 2x - 20$	
		$f(2) = -8 < 0$	
		$f(3) = 13 > 0$	½
		$f'(x) = 3x^2 + 2$	½
		$\therefore\ f'(2) = 14$	
		$\therefore$ Initial root $x_0 = 2$	
		$x_{n+1} = \frac{xf'(x) - f(x)}{f'(x)}$	
		$x_{n+1} = \frac{x(3x^2+2) - (x^3+2x-20)}{3x^2+2}$	
		$x_{n+1} = \frac{3x^3+2x-x^3-2x+20}{3x^2+2}$	
		$x_{n+1} = \frac{2x^3+20}{3x^2+2}$	
		$n = 0,1,2$	
		$x_1 = 2.571$	1
		$x_2 = 2.473$	1
		$x_3 = 2.469$	1
		--	
	f)	Find approximate value of $\sqrt[3]{100}$ by using Newton-Raphson method (Three iterations only)	**04**
	Ans	Let $x = \sqrt[3]{100}$	
		$\therefore x^3 = 100$	
		$\therefore x^3 - 100 = 0$	
		$\therefore\ f(x) = x^3 - 100$	
		$f(4) = -36 < 0$	
		$f(5) = 25 > 0$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	f)	$f'(x)=3x^2$	½
		Initial root $x_0=5$	
		$x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=4.667$	1
		$x_2=4.667-\dfrac{f(4.667)}{f'(4.667)}=4.642$	1
		$x_3=4.642-\dfrac{f(4.642)}{f'(4.642)}=4.642$	1
		OR	
		Let $x=\sqrt[3]{100}$	
		$\therefore x^3=100$	
		$\therefore x^3-100=0$	
		$\therefore f(x)=x^3-100$	
		$f(4)=-36<0$	
		$f(5)=25>0$	½
		$f'(x)=3x^2$	½
		Initial root $x_0=5$	
		$x_i=x-\dfrac{f(x)}{f'(x)}=x-\dfrac{x^3-100}{3x^2}$	
		$=\dfrac{3x^3-x^3+100}{3x^2}$	
		$=\dfrac{2x^3+100}{3x^2}$	
		$x_1=4.667$	1
		$x_2=4.642$	1
		$x_3=4.642$	1

		Attempt any FOUR of the following:	**20**
6.	a)	Differentiate $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to $\sec^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right)$	**04**
	Ans	Let $u=\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ and $v=\sec^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right)$	
		Put $x=\sin\theta\Rightarrow\sin^{-1}x=\theta$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	a)	$u=\cos^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$	
		$u=\cos^{-1}\left(2\sin\theta\cos\theta\right)$	½
		$u=\cos^{-1}\left(\sin 2\theta\right)$	
		$u=\cos^{-1}\left(\cos\left(\frac{\pi}{2}-2\theta\right)\right)$	½
		$u=\frac{\pi}{2}-2\theta$	
		$u=\frac{\pi}{2}-2\sin^{-1}x$	½
		$\frac{du}{dx}=0-2\frac{1}{\sqrt{1-x^2}}$	
		$\frac{du}{dx}=-\frac{2}{\sqrt{1-x^2}}$	½
		$v=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$	
		$v=\sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$	
		$v=\sec^{-1}\left(\frac{1}{\sqrt{\cos^2\theta}}\right)$	½
		$v=\sec^{-1}\left(\frac{1}{\cos\theta}\right)$	
		$v=\sec^{-1}\left(\sec\theta\right)$	
		$v=\theta$	
		$v=\sin^{-1}x$	
		$\frac{dv}{dx}=\frac{1}{\sqrt{1-x^2}}$	½
		$\therefore \frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{-\frac{2}{\sqrt{1-x^2}}}{\frac{1}{\sqrt{1-x^2}}}$	
		$\therefore \frac{du}{dv}=-2$	1
		OR	
		Let $u=\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ and $v=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	a)	Put $x=\cos\theta \Rightarrow \cos^{-1}x=\theta$	
		$u=\cos^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right)$	½
		$u=\cos^{-1}\left(2\cos\theta\sin\theta\right)$	½
		$u=\cos^{-1}\left(\sin 2\theta\right)$	½
		$u=\cos^{-1}\left(\cos\left(\frac{\pi}{2}-2\theta\right)\right)$	
		$u=\frac{\pi}{2}-2\theta$	
		$u=\frac{\pi}{2}-2\cos^{-1}x$	
		$\frac{du}{dx}=0-2\left(\frac{-1}{\sqrt{1-x^2}}\right)$	
		$\frac{du}{dx}=\frac{2}{\sqrt{1-x^2}}$	½
		$v=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$	
		$v=\sec^{-1}\left(\frac{1}{\sqrt{1-\cos^2\theta}}\right)$	
		$v=\sec^{-1}\left(\frac{1}{\sqrt{\sin^2\theta}}\right)$	
		$v=\sec^{-1}\left(\frac{1}{\sin\theta}\right)$	
		$v=\sec^{-1}\left(\cos ec\theta\right)$	½
		$v=\sec^{-1}\left(s\,ec\left(\frac{\pi}{2}-\theta\right)\right)$	½
		$v=\frac{\pi}{2}-\theta$	
		$v=\frac{\pi}{2}-\cos^{-1}x$	
		$\frac{dv}{dx}=\frac{1}{\sqrt{1-x^2}}$	½
		$\therefore \frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\frac{2}{\sqrt{1-x^2}}}{\frac{1}{\sqrt{1-x^2}}}$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	a)	$\therefore \dfrac{du}{dv}=2$	½
		--	
	b)	If $y=A\cos(\log x)+B\sin(\log x)$, prove that $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=0$	**04**
	Ans	$y=A\cos(\log x)+B\sin(\log x)$	
		$\dfrac{dy}{dx}=A\left(\dfrac{-\sin(\log x)}{x}\right)+B\left(\dfrac{\cos(\log x)}{x}\right)$	1
		$x\dfrac{dy}{dx}=-A\sin(\log x)+B\cos(\log x)$	
		$x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=-A\left(\dfrac{\cos(\log x)}{x}\right)+B\left(\dfrac{-\sin(\log x)}{x}\right)$	1
		$x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=-\left(A\cos(\log x)+B\sin(\log x)\right)$	1
		$x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=-y$	
		$x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=0$	1
		--	
	c)	Solve by Gauss-elimination method	**04**
		$x+2y+3z=14$, $3x+y+2z=11$, $2x+3y+z=11$	
	Ans	$x+2y+3z=14$ $3x+y+2z=11$ $2x+3y+z=11$	
		$x+2y+3z=14$ $6x+2y+4z=22$ $-\quad-\quad-\quad-$ $-5x-z=-8$ and $9x+3y+6z=33$ $2x+3y+z=11$ $-\quad-\quad-\quad-$ $7x+5z=22$	½ ½
		$-25x-5z=-40$ $\underline{7x+5z=22}$ $-18x=-18$	
		$\therefore x=1$	1
		$y=2$	1
		$z=3$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.		***Note:*** *In the above solution, first y is eliminated and then z is eliminated to find the value of x first. If in case the problem is solved by elimination of another unknown i. e., either y or z, appropriate marks to be given as per above scheme of marking.*	
	d)	Solve by Jacobi's method $10x+y+2z=13$, $3x+10y+z=14$, $2x+3y+10z=15$ (Three iterations only)	**04**
	Ans	$x=\frac{1}{10}(13-y-2z)$ $y=\frac{1}{10}(14-3x-z)$ $z=\frac{1}{10}(15-2x-3y)$	1
		Starting with $x_0=y_0=z_0=0$ $x_1=1.3$ $y_1=1.4$ $z_1=1.5$	1
		$x_2=0.86$ $y_2=0.86$ $z_2=0.82$	1
		$x_3=1.05$ $y_3=1.06$ $z_3=1.07$	1
		--	
	e)	Solve by using Gauss-Seidel method $6x+y+z=105$, $4x+8y+3z=155$, $5x+4y-10z=65$ (Two iterations only)	**04**
	Ans	$x=\frac{1}{6}(105-y-z)$ $y=\frac{1}{8}(155-4x-3z)$ $z=\frac{1}{-10}(65-5x-4y)$	1
		Starting with $y_0=z_0=0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	e)	$x_1 = 17.5$ $y_1 = 10.625$ $z_1 = 6.5$	1½
		$x_2 = 14.646$ $y_2 = 9.615$ $z_2 = 4.669$	1½
		--	
	f)	Solve by Gauss-Seidal method: $x+7y-3z=-22$, $5x-2y+3z=18$, $2x-y+6z=22$ (Two iterations only)	**04**
	Ans	$x=\frac{1}{5}(18+2y-3z)$ $y=\frac{1}{7}(-22-x+3z)$ $z=\frac{1}{6}(22-2x+y)$	1
		Starting with $y_0 = z_0 = 0$ $x_1 = 3.6$ $y_1 = -3.657$ $z_1 = 1.857$	1½
		$x_2 = 1.023$ $y_2 = -2.493$ $z_2 = 2.91$	1½
		--	
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* -- --	